

Circuit and Network Theory

LECTURE NOTES Semester : 3rd

Department of Electrical Engineering





ELECTRICAL CIRCUITS

UNIT-1

INTRODUCTION TO ELECTRICAL CIRCUITS

- Concept of Network and circuit
- Types of Elements
- Types of Sources
- Source Transformation
- R-L-C Parameters
- Voltage - Current relationships for Passive Elements (For different Input Signals : Square, Ramp, Saw tooth and Triangle)
- Kirchhoff's Laws



INTRODUCTION:

An Electric circuit is an interconnection of various elements in which there is at least one closed path in which current can flow. An Electric circuit is used as a component for any engineering system.

The performance of any electrical device or machine is always studied by drawing its electrical equivalent circuit. By simulating an electric circuit, any type of system can be studied for e.g., mechanical, hydraulic thermal, nuclear, traffic flow, weather prediction etc.

All control systems are studied by representing them in the form of electric circuits. The analysis, of any system can be learnt by mastering the techniques of circuit theory.

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Elements of an Electric circuit:

An Electric circuit consists of following types of elements.

Active elements:

Active elements are the elements of a circuit which possess energy of their own and can impart it to other element of the circuit.

Active elements are of two types

a) Voltage source

b) Current source

A Voltage source has a specified voltage across its terminals, independent of current flowing through it.

A current source has a specified current through it independent of the voltage appearing across it.

Passive Elements:

The passive elements of an electric circuit do not possess energy of their own. They receive energy from the sources. The passive elements are the resistance, the inductance and the capacitance. When electrical energy is supplied to a circuit element, it will respond in one and more of the following ways.

If the energy is consumed, then the circuit element is a pure resistor.

If the energy is stored in a magnetic field, the element is a pure inductor.

And if the energy is stored in an electric field, the element is a pure capacitor.

Linear and Non-Linear Elements.

Linear elements show the linear characteristics of voltage & current. That is its voltage-current characteristics are at all-times a straight-line through the origin.

For example, the current passing through a resistor is proportional to the voltage applied through its and the relation is expressed as $V \propto I$ or $V = IR$. A linear element or network is one which satisfies the principle of superposition, i.e., the principle of homogeneity and additivity.

Resistors, inductors and capacitors are the examples of the linear elements and their properties do not change with a change in the applied voltage and the circuit current.

Non linear element's V-I characteristics do not follow the linear pattern i.e. the current passing through it does not change linearly with the linear change in the voltage across it. Examples are the semiconductor devices such as diode, transistor.

Bilateral and Unilateral Elements:

An element is said to be bilateral, when the same relation exists between voltage and current for the current flowing in both directions.

Ex: Voltage source, Current source, resistance, inductance & capacitance.

The circuits containing them are called bilateral circuits.

An element is said to be unilateral, when the same relation does not exist between voltage and current when current flowing in both directions. The circuits containing them are called unilateral circuits.

Ex: Vacuum diodes, Silicon Diodes, Selenium Rectifiers etc.

Lumped and Distributed Elements

Lumped elements are those elements which are very small in size & in which simultaneous actions takes place. Typical lumped elements are capacitors, resistors, inductors.

Distributed elements are those which are not electrically separable for analytical purposes.

For example a transmission line has distributed parameters along its length and may extend for hundreds of miles.

The circuits containing them are called unilateral circuits.

Types of Sources:

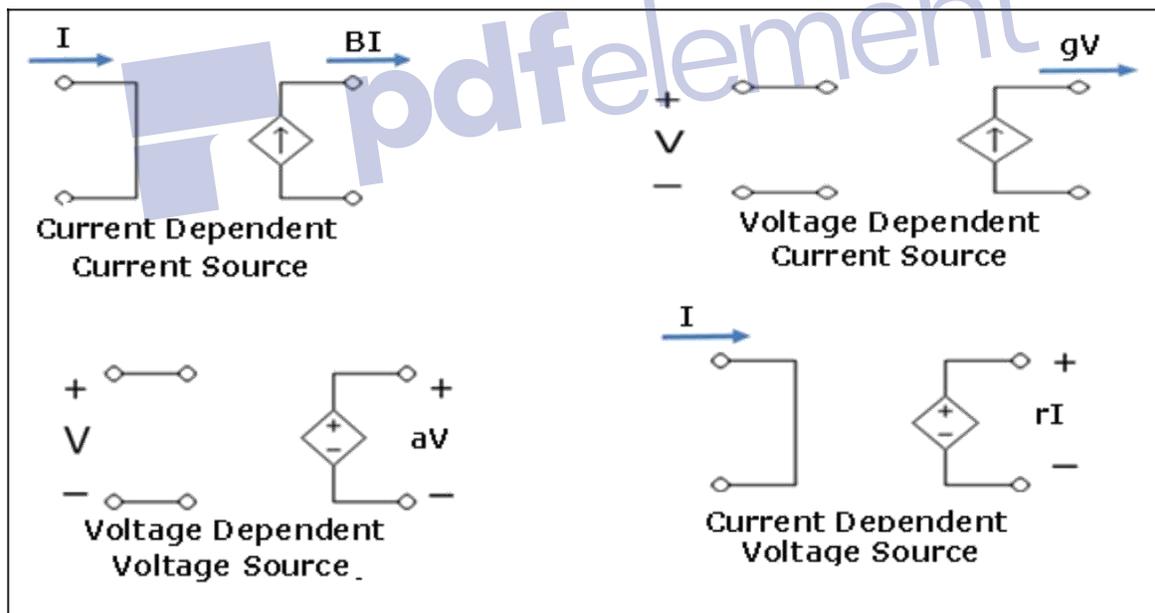
Independent & Dependent sources:

If the voltage of the voltage source is completely independent source of current and the current of the current source is completely independent of the voltage, then the sources are called as independent sources.

The special kind of sources in which the source voltage or current depends on some other quantity in the circuit which may be either a voltage or a current anywhere in the circuit are called Dependent sources or Controlled sources.

There are four possible dependent sources:

- Voltage dependent Voltage source
- Current dependent Current source
- Voltage dependent Current source
- Current dependent Voltage source

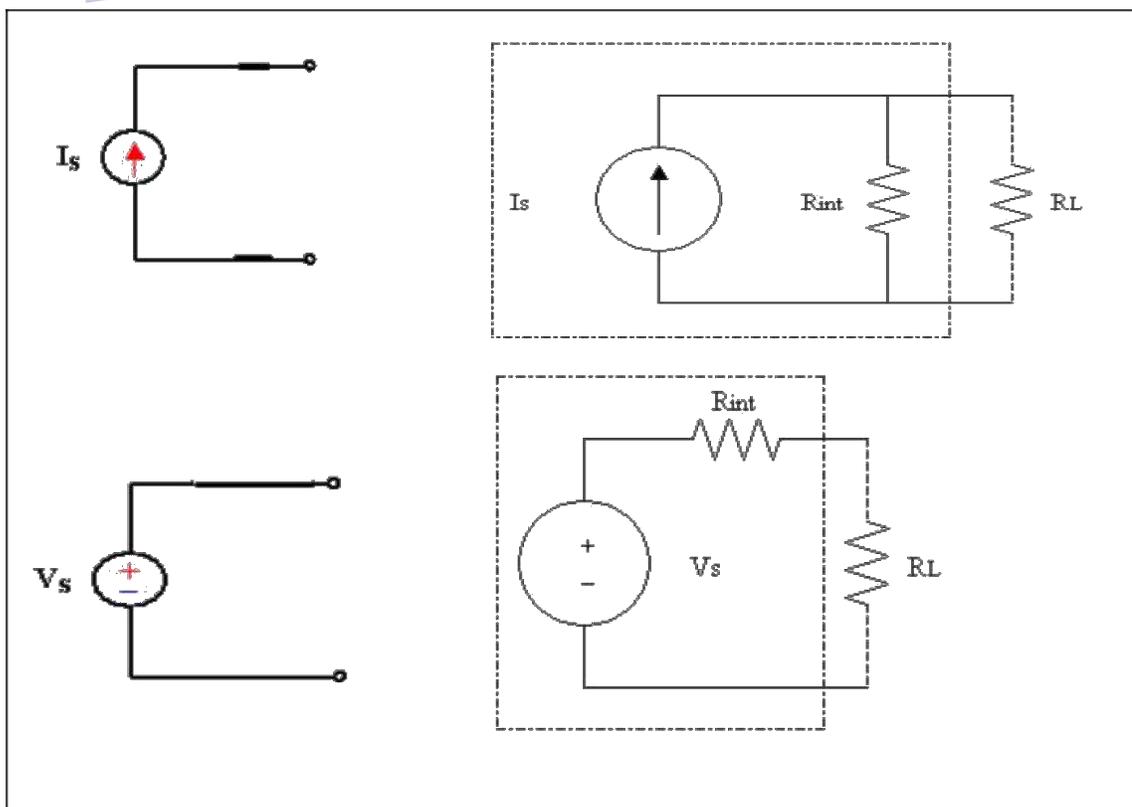


The constants of proportionalities are written as B , g , a , r in which B & a has no units, r has units of ohm & g units of mhos.

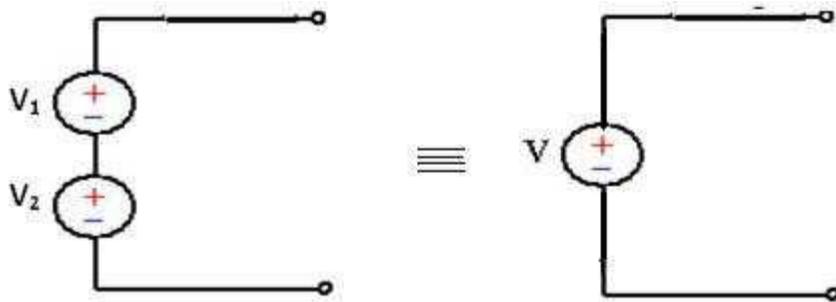
Independent sources actually exist as physical entities such as battery, a dc generator & an alternator. But dependent sources are used to represent electrical properties of electronic devices such as OPAMPS & Transistors.

Ideal & Practical sources:

1. An ideal voltage source is one which delivers energy to the load at a constant terminal voltage, irrespective of the current drawn by the load.
2. An ideal current source is one, which delivers energy with a constant current to the load, irrespective of the terminal voltage across the load.
3. A Practical voltage source always possesses a very small value of internal resistance r . The internal resistance of a voltage source is always connected in series with it & for a current source; it is always connected in parallel with it. As the value of the internal resistance of a practical voltage source is very small, its terminal voltage is assumed to be almost constant within a certain limit of current flowing through the load.
4. A practical current source is also assumed to deliver a constant current, irrespective of the terminal voltage across the load connected to it.



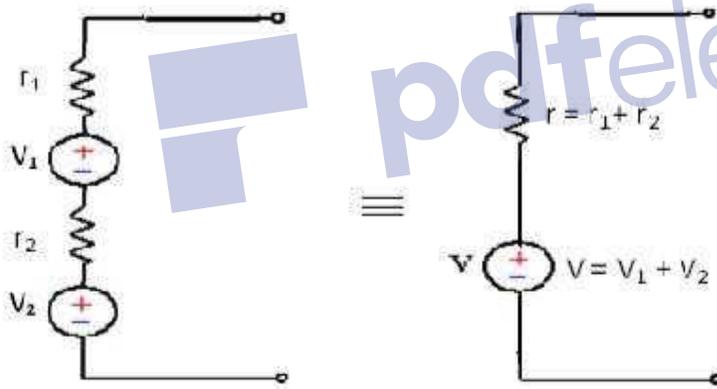
Ideal voltage source connected in series:



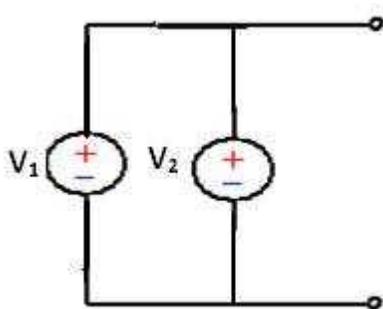
The equivalent single ideal voltage source is given by $V = V_1 + V_2$

Any number of ideal voltage sources connected in series can be represented by a single ideal voltage source taking into account the polarities connected together for consideration.

Practical voltage source connected in series:



Ideal voltage source connected in parallel:



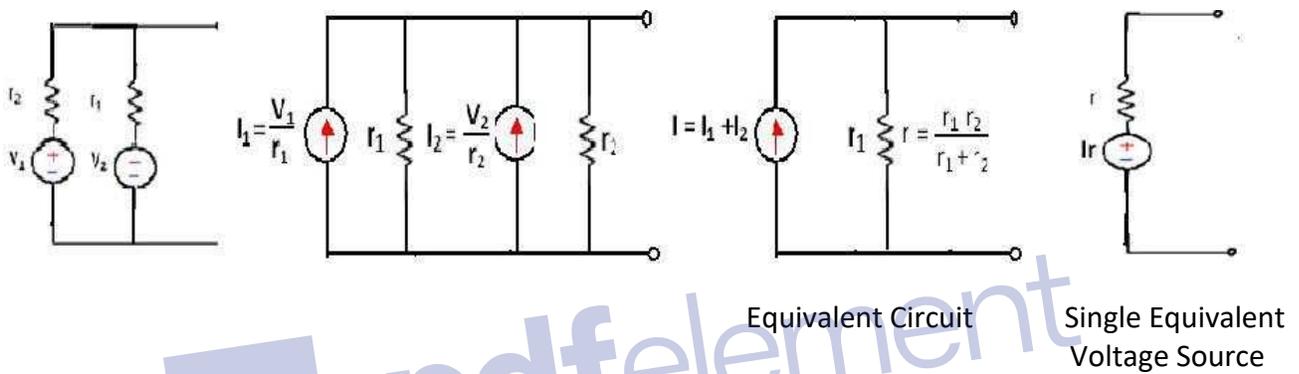
When two ideal voltage sources of emf's V_1 & V_2 are connected in parallel, what voltage appears across its terminals is ambiguous.

Hence such connections should not be made.

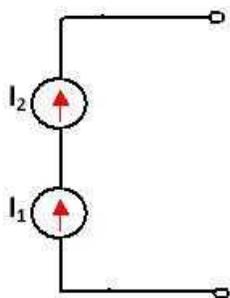
However if $V_1 = V_2 = V$, then the equivalent voltage source is represented by V .

In that case also, such a connection is unnecessary as only one voltage source serves the purpose.

Practical voltage sources connected in parallel:



Ideal current sources connected in series:

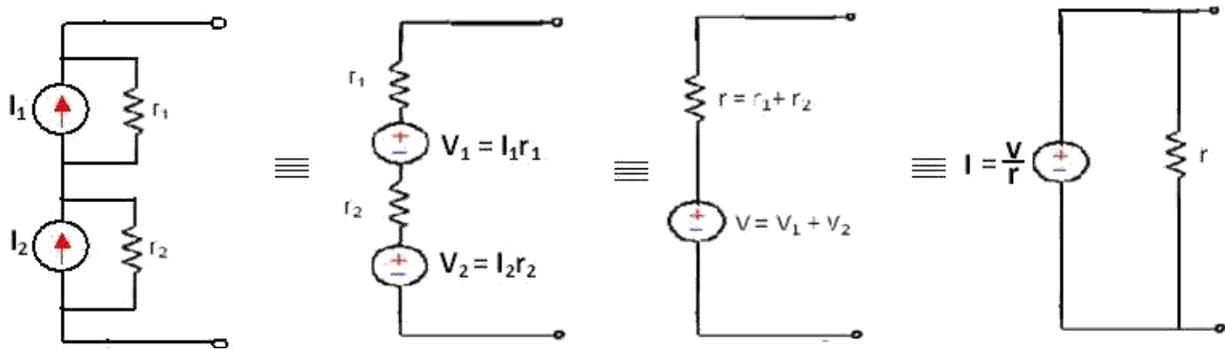


When ideal current sources are connected in series, what current flows through the line is ambiguous. Hence such a connection is not permissible.

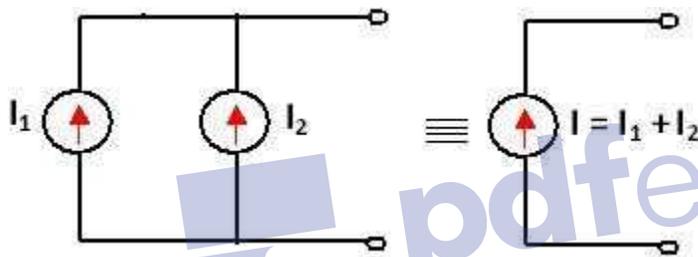
However, if $I_1 = I_2 = I$, then the current in the line is I .

But, such a connection is not necessary as only one current source serves the purpose.

Practical current sources connected in series:

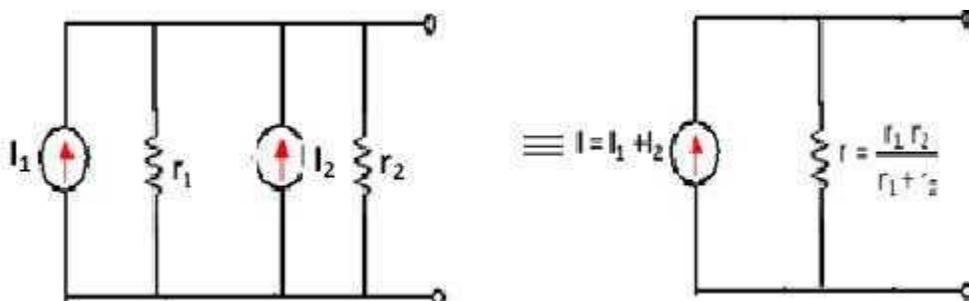


Ideal current sources connected in parallel



Two ideal current sources in parallel can be replaced by a single equivalent ideal current source.

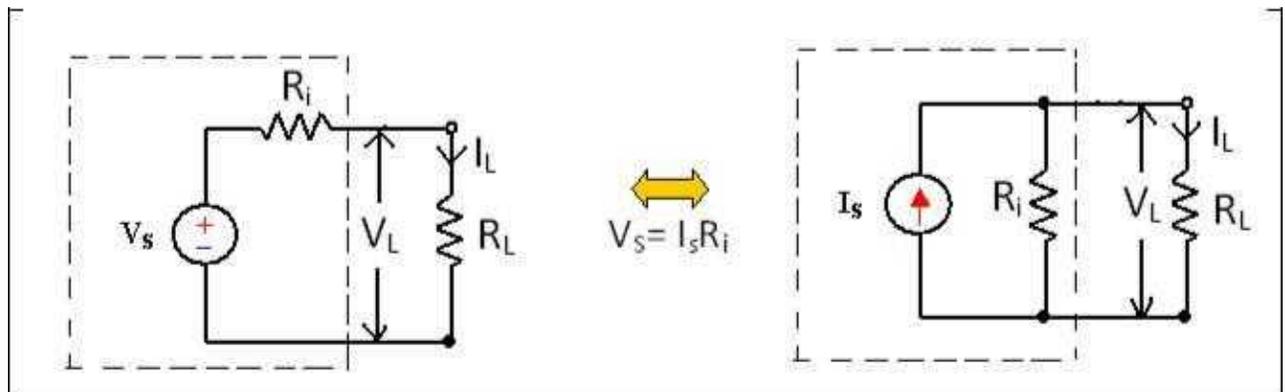
Practical current sources connected in parallel



Source transformation:

A current source or a voltage source drives current through its load resistance and the magnitude of the current depends on the value of the load resistance.

Consider a practical voltage source and a practical current source connected to the same load resistance R_L as shown in the figure



R_i 's in figure represents the internal resistance of the voltage source V_S and current source I_S .

Two sources are said to be identical, when they produce identical terminal voltage V_L and load current I_L .

The circuit in figure represents a practical voltage source & a practical current source respectively, with load connected to both the sources.

The terminal voltage V_L and load current I_L across their terminals are same.

Hence the practical voltage source & practical current source shown in the dotted box of figure are equal.

The two equivalent sources should also provide the same open circuit voltage & short circuit current.

From fig (a)

$$I_L = \frac{V_S}{R + R_L}$$

From fig (b)

$$I_L = I \frac{r}{R + R_L}$$

$$\frac{V_S}{R + R_L} = I$$

$$V_S = IR \quad \text{or} \quad I = \frac{V_S}{R}$$

Hence a voltage source V_s in series with its internal resistance R can be converted into a current source $I = \frac{V_s}{R}$, with its internal resistance R connected in parallel with it. Similarly a current source I in parallel with its internal resistance R can be converted into a voltage source $V = IR$ in series with its internal resistance R .

R-L-C Parameters:

1. Resistance:

Resistance is that property of a circuit element which opposes the flow of electric current and in doing so converts electrical energy into heat energy.

It is the proportionality factor in ohm's law relating voltage and current.

Ohm's law states that the voltage drop across a conductor of given length and area of cross section is directly proportional to the current flowing through it.

$$R \propto l$$

$$V = Ri$$

$$i = \frac{V}{R} = GV$$

Where the reciprocal of resistance is called conductance G . The unit of resistance is ohm and the unit of conductance is mho or Siemens.

When current flows through any resistive material, heat is generated by the collision of electrons with other atomic particles. The power absorbed by the resistor is converted to heat and is given by the expression

$$P = vi = i^2R \text{ where } i \text{ is the resistor in amps, and } v \text{ is the voltage across the resistor in volts.}$$

Energy lost in a resistance in time t is given by

$$W = \int_0^t p \, dt = pt = i^2Rt = \frac{v^2}{R} t$$

2. Inductance:

Inductance is the property of a material by virtue of which it opposes any change of magnitude and direction of electric current passing through conductor. A wire of certain length, when twisted into a coil becomes a basic conductor. A change in the magnitude of the current changes the electromagnetic field.

Increase in current expands the field & decrease in current reduces it. A change in current produces change in the electromagnetic field. This induces a voltage across the coil according to Faradays laws of Electromagnetic Induction.

$$\text{Induced Voltage } V = L \frac{di}{dt}$$

V = Voltage across inductor in volts

I = Current through inductor in amps

$$di = \frac{1}{L} v dt$$

Integrating both sides,

$$\int_0^t di = \frac{1}{L} \int_0^t v dt$$

$$\text{Power absorbed by the inductor } P = VI = Li \frac{di}{dt}$$

Energy stored by the inductor

$$W = \int_0^t P dt = \int_0^t Li \frac{di}{dt} dt = \frac{Li^2}{2}$$

$$W = \frac{Li^2}{2}$$

Conclusions:

$$1) V = L \frac{di}{dt}$$

The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to dc.

2) For minute change in current within zero time ($dt = 0$) gives an infinite voltage across the inductor which is physically not at all feasible. In an inductor, the current cannot change abruptly. An inductor behaves as open circuit just after switching across dc voltage.

3) The inductor can store finite amount of energy, even if the voltage across the inductor is zero.

4) A pure inductor never dissipates energy, it only stores it. Hence it is also called as a non-dissipative passive element. However, physical inductor dissipates power due to internal resistance.

Ex: The current in a 2H inductor raises at a rate of 2A/s .Find the voltage across the inductor the energy stored in the magnetic field at after 2sec.

Sol:

$$V = L \frac{di}{dt}$$
$$= 2 \times 2 = 4V$$



$$W = \frac{1}{2} Li^2 = \frac{1}{2} \times 2 \times (4)^2 = 16 \text{ J}$$

3. Capacitance:

- 1) A capacitor consists of two metallic surfaces or conducting surfaces separated by a dielectric medium.
- 2) It is a circuit element which is capable of storing electrical energy in its electric field.
- 3) Capacitance is its capacity to store electrical energy.
- 4) Capacitance is the proportionality constant relating the charge on the conducting plates to the potential.

Charge on the capacitor $q \propto V$

$$q = CV$$

Where 'C' is the capacitance in farads, if q is charge in coulombs and V is the potential difference across the capacitor in volts.

The current flowing in the circuit is rate of flow of charge

$$i = \frac{dq}{dt} = C \frac{dv}{dt} \quad \therefore i = C \frac{dv}{dt}$$

The capacitance of a capacitor depends on the dielectric medium & the physical dimensions. For a parallel plate capacitor, the capacitance

$$C = \frac{\epsilon A}{D} = \epsilon_0 \epsilon_r \frac{A}{D}$$

A is the surface area of plates

D is the separation between plates

ϵ is the absolute permeability of medium ϵ_0 is the absolute permeability of free

space ϵ_r is the relative permeability of medium

$$i = \frac{dq}{dt} = C \frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{i}{C}$$

$$V = \frac{1}{C} \int i dt$$

The power absorbed by the capacitor $P = vi = vC \frac{dv}{dt}$

$$\text{Energy stored in the capacitor } W = \int_0^t P dt = \int_0^t VC \frac{dv}{dt} dt$$

$$= C \int_0^t v dv = \frac{1}{2} C v^2 \text{ Joules}$$

This energy is stored in the electric field set up by the voltage across capacitor.

Conclusions:

1. The current in a capacitor is zero, if the voltage across it is constant, that means the capacitor acts as an open circuit to dc
2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible.
 - ⚠ In a fixed capacitor, the voltage cannot change abruptly
 - ⚠ A capacitor behaves as short circuit just after switching across dc voltage.
3. The capacitor can store a finite amount of energy, even if the current through it is zero.
4. A pure capacitor never dissipates energy but only stores it hence it is called non-dissipative element.

Kirchhoff's Laws:

Kirchhoff's laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter.

Kirchhoff's laws, two in number, are particularly useful in determining the equivalent resistance of a complicated network of conductors and for calculating the currents flowing in the various conductors.

1. Kirchhoff's Current Law (KCL)

In any electrical network, the algebraic sum of the currents meeting at a point (or junction) is Zero.

That is the total current **entering** a junction is equal to the total current **leaving** that junction.

Consider the case of a network shown in Fig (a).

$$I_1 + (-I_2) + I_3 + (+I_4) + (-I_5) = 0$$

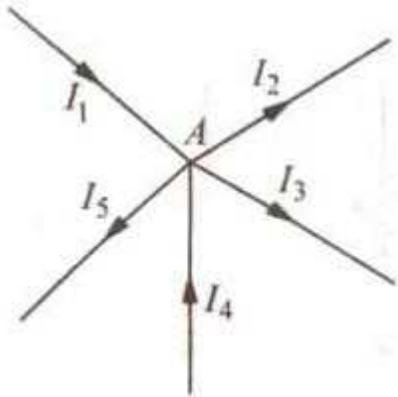
$$I_1 + I_4 - I_2 - I_3 - I_5 = 0$$

Or

$$I_1 + I_4 = I_2 + I_3 + I_5$$

Or

Incoming currents = Outgoing currents



2. Kirchhoff's Mesh Law or Voltage Law (KVL)

In any electrical network, the algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.f.'s. in that path is zero.

That is, $\sum IR + \sum e.m.f = 0$ round a mesh

It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.

That is, if we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started.

Hence, it means that all the sources of emf met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

Determination of Voltage Sign

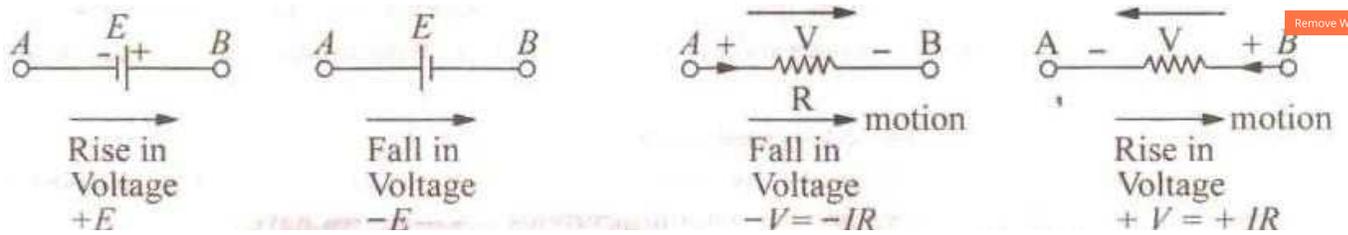
In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs.

(a) Sign of Battery E.M.F.

A *rise* in voltage should be given a + ve sign and a *fall* in voltage a -ve sign. That is, if we go from the -ve terminal of a battery to its +ve terminal there is a *rise* in potential, hence this voltage should be given a + ve sign.

And on the other hand, we go from +ve terminal to -ve terminal, then there is a *fall* in potential, hence this voltage should be preceded by a -ve sign.

The sign of the battery e.m.f is independent of the direction



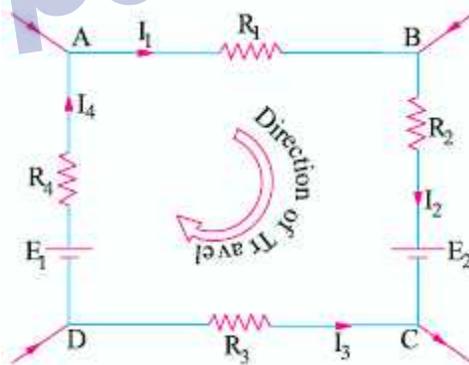
of the current through that branch.

(b) Sign of IR Drop

Now, take the case of a resistor (Fig. 2.4). If we go through a resistor in the *same* direction as the current, then there is a fall in potential because current flows from a higher to a lower potential..

Hence, this voltage fall should be taken -ve. However, if we go in a direction opposite to that of the current, then there is a *rise* in voltage. Hence, this voltage rise should be given a positive sign.

Consider the closed path ABCDA in Fig .



As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs :

- I_1R_1 is - ve (fall in potential)
- I_2R_2 is - ve (fall in potential)
- I_3R_3 is + ve (rise in potential)
- I_4R_4 is - ve (fall in potential)
- E_2 is - ve (fall in potential)
- E_1 is + ve (rise in potential)

Using Kirchhoff's voltage law, we get

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

Or $I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$

Assumed Direction of Current:

In applying Kirchhoff's laws to electrical networks, the direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the question, the current will be found to have a minus sign.

If the answer is positive, then assumed direction is the same as actual direction. However, the important point is that once a particular direction has been assumed, the same should be used throughout the solution of the question.

Kirchhoff's laws are applicable both to d.c. and a.c. voltages and currents. However, in the case of alternating currents and voltages, any e.m.f. of self-inductance or that existing across a capacitor should be also taken into account.





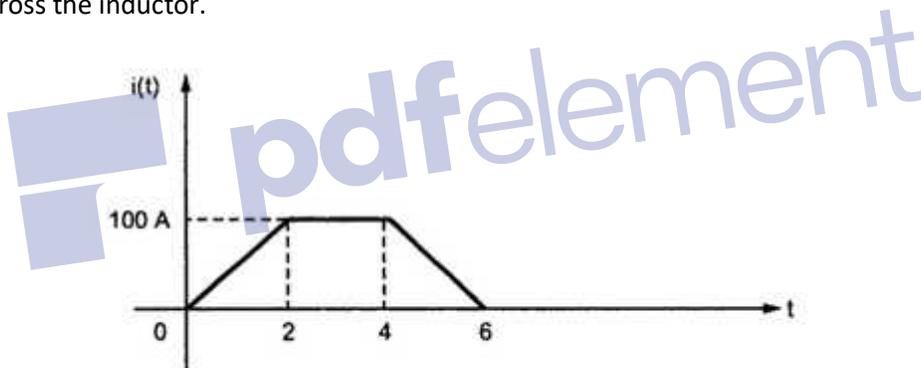
VOLTAGE-CURRENT RELATIONSHIPS FOR PASSIVE ELEMENTS

The Three Passive Elements are Resistance, Inductance and Capacitance. The behavior of these three elements along with the respective voltage-current relationship is given in the table.

Element	Basic Relation	Voltage across, If Current Known	Current through, If Voltage Known
R	$R = \frac{V}{I}$	$V_R(t) = R i_R(t)$	$i_R(t) = \frac{V_R(t)}{R}$
L	$L = \frac{N\Phi}{I}$	$V_L(t) = L \frac{di(t)}{dt}$	$i_L(t) = \frac{1}{L} \int_{-\infty}^t V(t) dt$
C	$C = \frac{Q}{V}$	$V_C(t) = \frac{1}{C} \int_{-\infty}^t i_C(t) dt$	$i_C(t) = C \frac{dv(t)}{dt}$

Table

1. A current waveform flowing through an inductor of 1mH is shown in the figure. Obtain and sketch the waveform of voltage across the inductor.



Solution:

From the given waveform,

For $0 < t < 2$, $i(t)$ is a straight line of slope $= (100/2) = 50$

Therefore, $i(t) = 50t$ and $\frac{di(t)}{dt} = 50$

For $2 < t < 4$, $i(t) = 100$ and $\frac{di(t)}{dt} = 0$

For $4 < t < 6$, $i(t)$ is a straight line of slope $= -(100/2) = -50$

Therefore, $i(t) = -50t$ and $\frac{di(t)}{dt} = -50$

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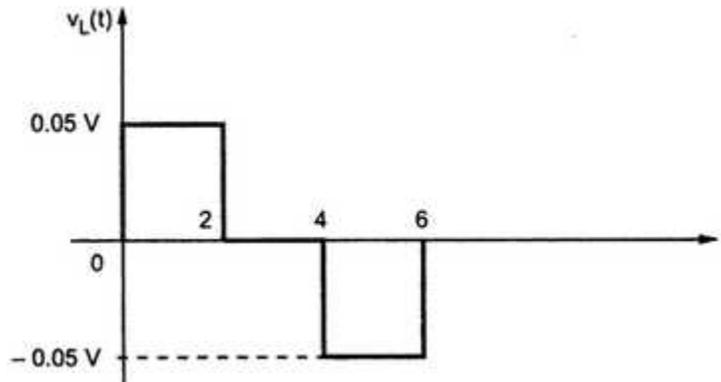
Now, $V_L(t) = L \frac{di(t)}{dt}$

$$= 1 \cdot 10^{-3} \cdot 50 = 0.05 \text{ V} \quad 0 < t < 2$$

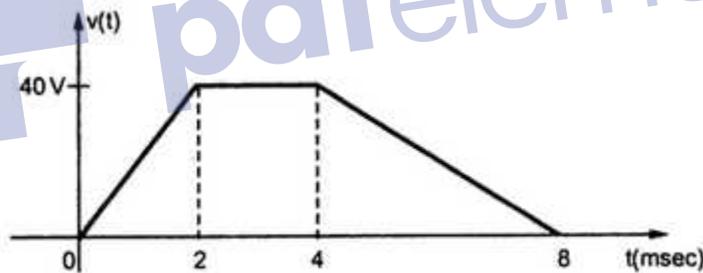
$$= 1 \cdot 10^{-3} \cdot 0 = 0 \text{ V} \quad 2 < t < 4$$

$$= 1 \cdot 10^{-3} \cdot (-50) = -0.05 \text{ V} \quad 4 < t < 6$$

The voltage waveform is shown in following figure.



2. A 0.5 μ F capacitor has voltage waveform $v(t)$ as shown in following figure, plot $i(t)$ as function of t ?



Solution:

From the given waveform,

For $0 < t < 2$, $v(t)$ is a ramp of slope $= (40/2) = 20$

Therefore $v(t) = 20t$

Therefore $i(t) = C \frac{dv(t)}{dt} = 0.5 \cdot 10^{-6} \cdot 20 = 1 \cdot 10^{-5} \text{ A} = 10 \mu\text{A}$

For $2 < t < 4$, $v(t)$ is constant

Therefore $v(t) = 40 \text{ V}$

ELECTRICAL CIRCUITS



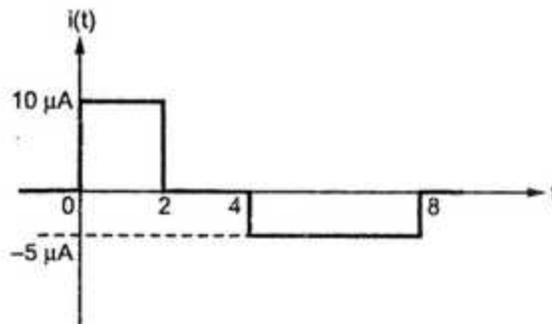
Therefore
$$i(t) = C \frac{dv(t)}{dt} = 0.5 \times 10^{-6} \times 0 = 0 \text{ A}$$

For $4 < t < 8$, $v(t)$ is a ramp with slope $= \frac{0-40}{8-4} = -10$

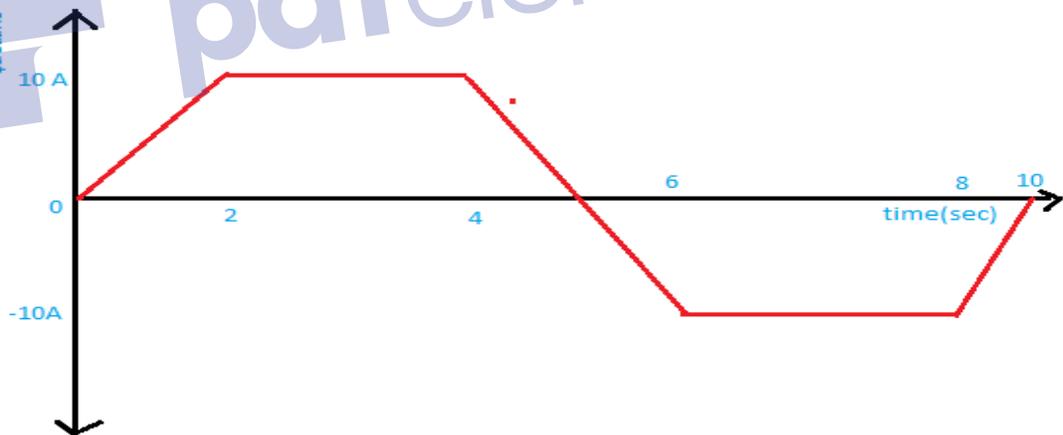
Therefore
$$v(t) = -10t + 80 \quad (\text{According to straight line equation i.e. } y = mx + c)$$

Therefore
$$i(t) = C \frac{dv(t)}{dt} = 0.5 \times 10^{-6} \times (-10) = -5 \mu\text{A}$$

The current waveform is shown in following figure



3. A Pure Inductance Of 3mh Carries A Current Of The Waveform Shown In Fig. Sketch The Waveform Of $V(t)$ And $P(t)$. Determine The Average Value Of Power



Fig

Solution:

$$i(t) = 5t \text{ for } 0 < t < 2$$

$$i(t) = 10 \text{ for } 2 < t < 4$$

$$i(t) = -10t + 50 \text{ for } 4 < t < 6$$

$$i(t) = -10 \text{ for } 6 < t < 8$$

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$$i(t) = 5t - 50 \text{ for } 8 < t < 10$$

$$\begin{aligned} \text{For } 0 < t < 2, \quad V_L(t) &= L \frac{di(t)}{dt} \\ &= 3 * 10^{-3} \frac{d(5t)}{dt} \\ &= 15 * 10^{-3} \text{v} \end{aligned}$$

$$\begin{aligned} \text{For } 2 < t < 4, \quad V_L(t) &= L \frac{di(t)}{dt} \\ &= 3 * 10^{-3} \frac{d(10)}{dt} \\ &= 0 \text{v} \end{aligned}$$

$$\begin{aligned} \text{For } 4 < t < 6, \quad V_L(t) &= L \frac{di(t)}{dt} \\ &= 3 * 10^{-3} \frac{d(-10t + 50)}{dt} \\ &= -30 * 10^{-3} \text{v} \end{aligned}$$

$$\begin{aligned} \text{For } 6 < t < 8, \quad V_L(t) &= L \frac{di(t)}{dt} \\ &= 3 * 10^{-3} \frac{d(-10)}{dt} \\ &= 0 \text{v} \end{aligned}$$

$$\begin{aligned} \text{For } 8 < t < 10, \quad V_L(t) &= L \frac{di(t)}{dt} \\ &= 3 * 10^{-3} \frac{d(5t - 50)}{dt} \\ &= 15 * 10^{-3} \text{v} \end{aligned}$$

The sketch of $v(t)$ is shown in fig.

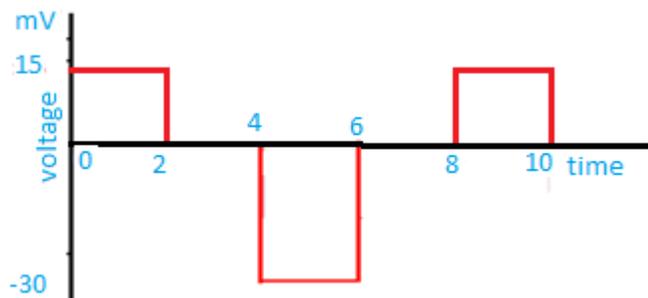


Fig.



For $0 < t < 2$,

$$P(t) = v(t)i(t) \\ = 75 \cdot 10^{-3}t \text{ W}$$

For $2 < t < 4$,

$$P(t) = v(t)i(t) \\ = 0 \text{ W}$$

For $4 < t < 6$,

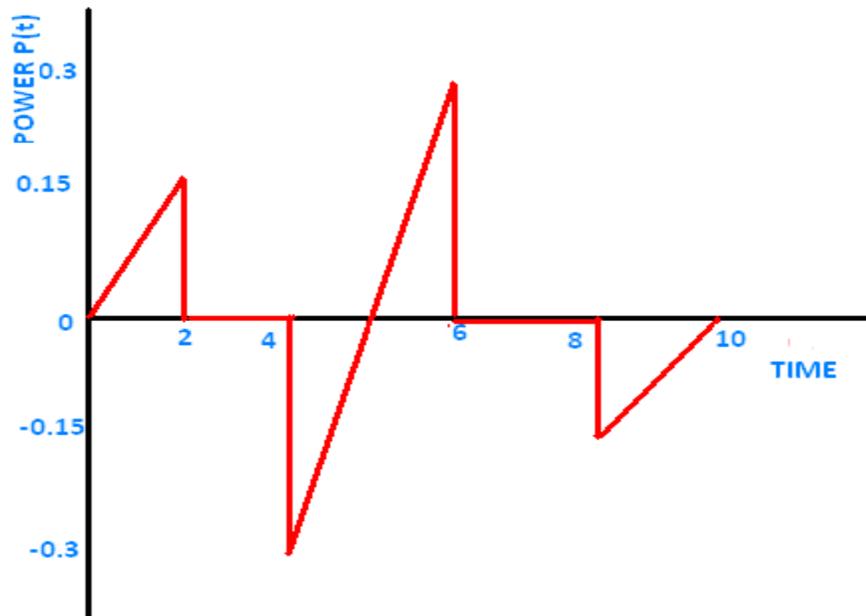
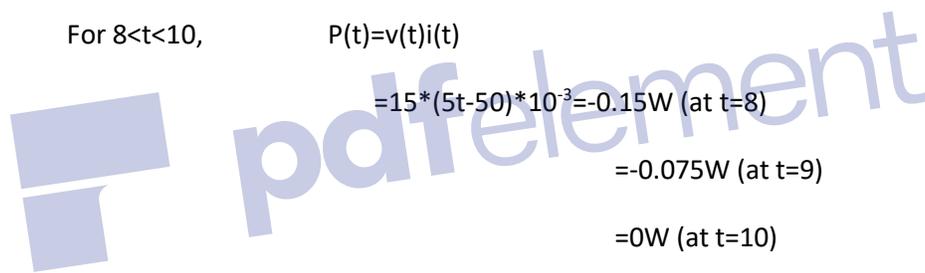
$$P(t) = v(t)i(t) \\ = -30 \cdot (-10t + 50) \cdot 10^{-3} = -0.3 \text{ W (at } t=4) \\ = 0 \text{ W (at } t=5) \\ = 0.3 \text{ W (at } t=6)$$

For $6 < t < 8$,

$$P(t) = v(t)i(t) \\ = 0 \text{ W}$$

For $8 < t < 10$,

$$P(t) = v(t)i(t) \\ = 15 \cdot (5t - 50) \cdot 10^{-3} = -0.15 \text{ W (at } t=8) \\ = -0.075 \text{ W (at } t=9) \\ = 0 \text{ W (at } t=10)$$

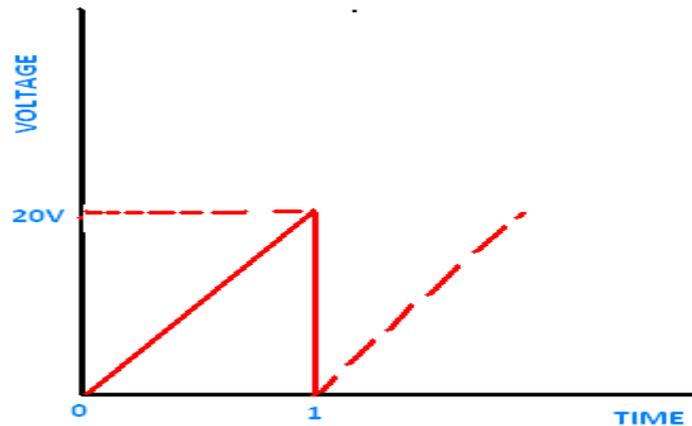


ELECTRICAL CIRCUITS



4. Draw the waveforms for current, power for the following elements if a voltage input shown in figure is applied to these elements.

- i. $R=1 \text{ OHM}$
- ii. $L=1\text{H}$
- iii. $C=1\text{F}$



SOLUTION:

From the figure, $v(t)$ is a straight line with slope $= \frac{20-0}{1-0} = 20$, For $0 < t < 1$

Therefore $v(t) = 20t$

i. **R=1 OHM**

The voltage and current relation of a resistor is given by, $v(t) = R i(t)$

$$i(t) = 20t/1 = 20t$$

Hence,

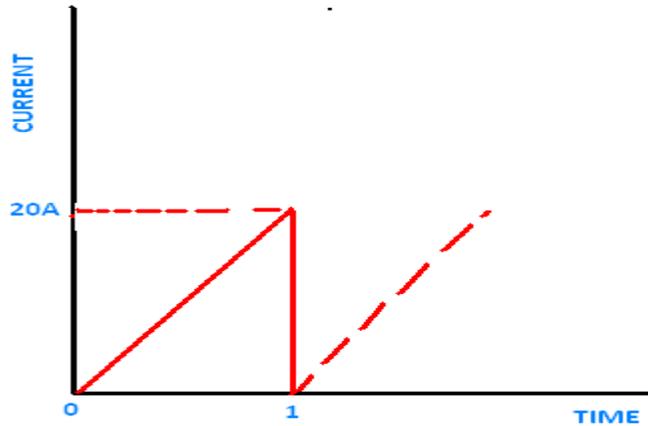
When $t=0$, $i(t)=0\text{A}$

When $t=0.5$, $i(t)=10\text{A}$

When $t=1$, $i(t)=20\text{A}$

Therefore the current waveform for the above values of t and $i(t)$ is shown in figure below

ELECTRICAL CIRCUITS



Power, $p(t) = v(t)i(t)$

$$= 20t * 20t = 400t^2 \text{ W}$$

Hence,

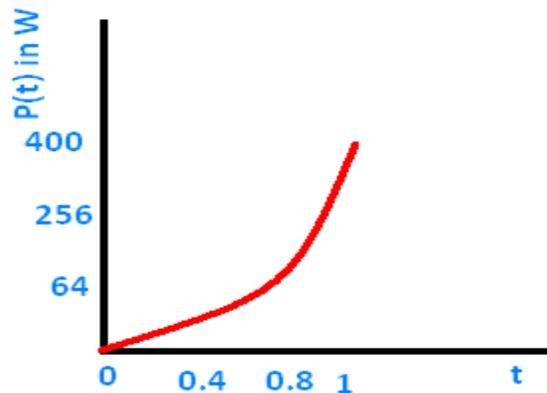
When $t=0$, $p(t) = v(t)i(t) = 0 \text{ W}$

When $t=0.4$, $p(t) = v(t)i(t) = 64 \text{ W}$

When $t=0.8$, $p(t) = v(t)i(t) = 256 \text{ W}$

When $t=1$, $p(t) = v(t)i(t) = 400 \text{ W}$

Therefore the power wave form for the above values of t and $p(t)$ is shown in below figure.



ii. **L=1 H**

The voltage and current relation of an inductor is given by,

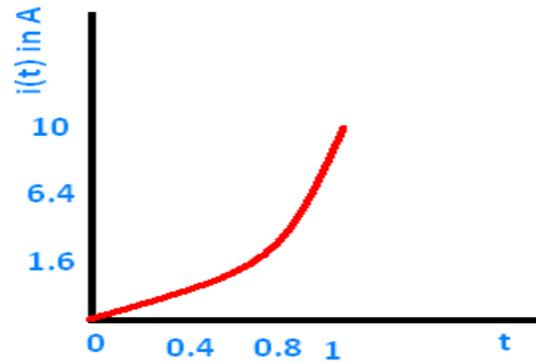
$$i(t) = \frac{1}{L} \int_{-\infty}^t V(t) dt$$

$$i(t) = \frac{1}{L} \left[\int_{-\infty}^0 V(t) dt + \int_0^t V(t) dt \right]$$



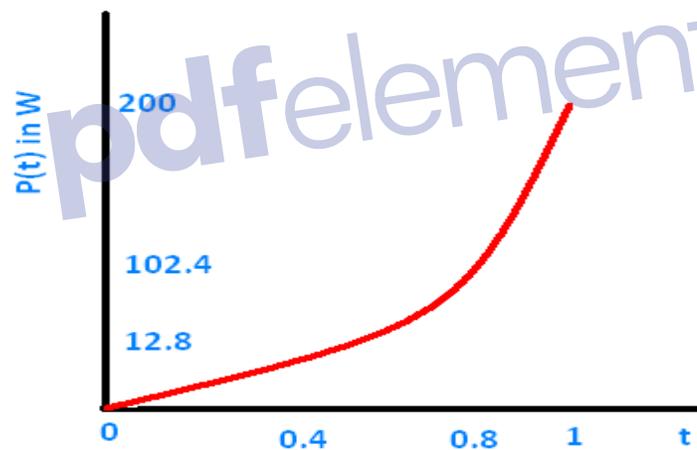
$$i(t) = 0 + \int_0^t V(t) dt = \int_0^t 20t dt = 10t^2$$

Therefore the current waveform is shown in below figure.



$$\text{Power, } p(t) = v(t)i(t) = 20t * 10t^2 = 200t^3 \text{ W}$$

Therefore the power waveform is shown in below figure



iii. **C=1 F**

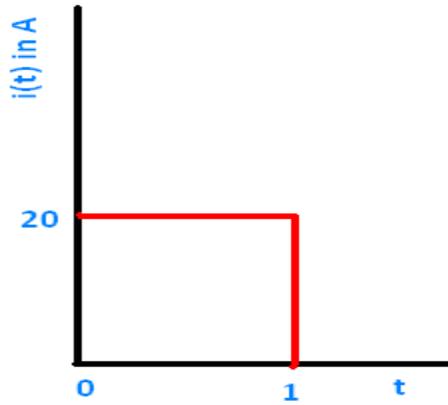
The voltage and current relation of a inductor is given by

$$i(t) = C \frac{dv(t)}{dt}$$

$$i(t) = 1 * \frac{d(20t)}{dt} = 20A$$

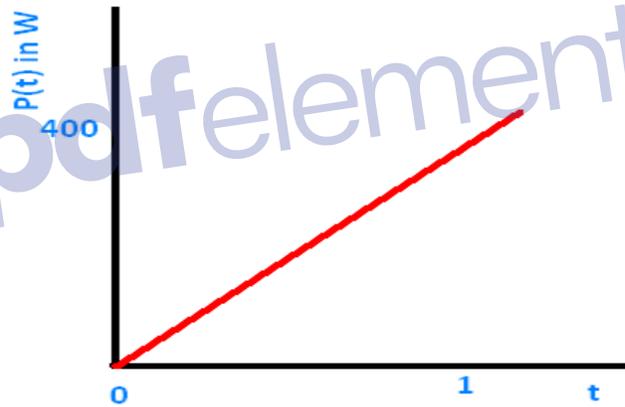


Therefore the current waveform is shown in below figure



$$\text{Power, } p(t) = v(t)i(t) = 20t * 20 = 400t \text{ W}$$

Therefore the power waveform is shown in below figure.



UNIT-II**NETWORK ANALYSIS**

- **Introduction**
- **Network Reduction Techniques**
- **Resistive Networks, Inductive Networks and Capacitive Networks**
- **Series, Parallel, and Series Parallel Connections**
- **Star to Delta and Delta to Star Transformations**
- **Mesh Analysis and Super Mesh for DC excitation**
- **Nodal Analysis and Super Node for DC excitation**
- **Network Topology Definitions: Graph, Tree, and Basic Tie-set, Basic Cut-set Matrices for planar Networks**



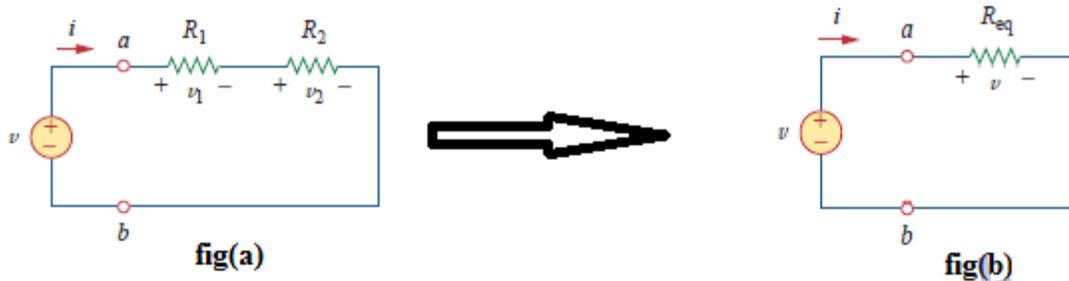
Introduction:

A network is a collection of interconnected electrical components. In general, the electrical networks are made to exchange the energy between different elements. These electrical networks can be constructed either by using Resistors or Inductors or Capacitors or combination of these elements. Network analysis is the process of finding the voltage response or the current response for any element in the network by using the available techniques.

Network Reduction Techniques:

Series Connection of Resistors:

Two or more resistors in a circuit are said to be in series when the current flowing through all the resistors is the same.



Consider the circuit in fig(a), where two resistors R_1 and R_2 are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, \dots \dots \dots (1)$$

$$v_2 = iR_2, \dots \dots \dots (2)$$

If we apply KVL to the loop fig(b), we have

$$-v + v_1 + v_2 = 0, \dots \dots \dots (3)$$

$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$i = v / (R_1 + R_2) \dots \dots \dots (4)$$

$$v = i R_{eq}, \dots \dots \dots (5)$$

Implying that the two resistors can be replaced by an equivalent resistor; that is,

$$R_{eq} = R_1 + R_2$$

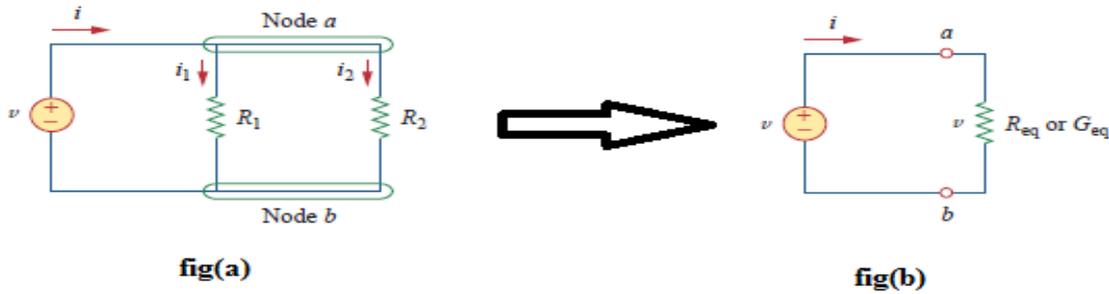
Note: The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

If "n" resistors are in series, $R_{eq} = R_1 + R_2 + \dots + R_n$

If "n" resistors of same value are in series, $R_{eq} = nR$

Parallel Connection of Resistors:

Two or more resistors in a circuit are said to be in Parallel when all the resistors are connected to the same nodes and the same voltage is appearing across all these elements.



Consider the circuit in fig(a), where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

$$v = i_1 R_1 = i_2 R_2 \dots \dots \dots (1)$$

$$i_1 = v/R_1, \quad i_2 = v/R_2 \dots \dots \dots (2)$$

Applying KCL at node *a* gives the total current *i* as

$$i = i_1 + i_2 \dots \dots \dots (3)$$

Substituting Eq. (2) into Eq. (3), we get

$$i = v/R_1 + v/R_2 = v(1/R_1 + 1/R_2) = v/Req$$

$$1/Req = 1/R_1 + 1/R_2$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Note: The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

If "n" resistors are in parallel, $1/Req = 1/R_1 + 1/R_2 + \dots + 1/R_n$

If "n" resistors of same value are in parallel, $Req = R/n$

It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. The equivalent conductance for *N* resistors in parallel is

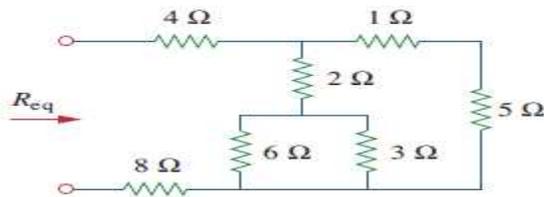
$$G_{eq} = G_1 + G_2 + G_3 + \dots + G_N$$

where $G_{eq} = 1/R_{eq}, G_1 = 1/R_1, G_2 = 1/R_2, G_3 = 1/R_3, \dots, G_N = 1/R_N.$

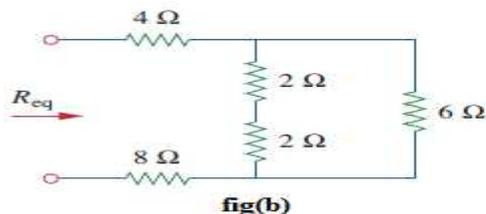
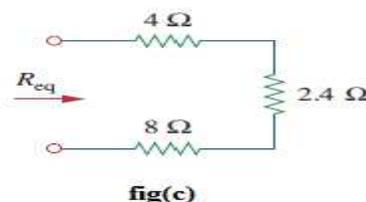
Note: The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

Example Problems:

1) Find the Req for the circuit shown in below figure.

**fig(a)**

Solution:

**fig(b)****fig(c)**

To get Req we combine resistors in series and in parallel. The 6 ohms and 3 ohms resistors are in parallel, so their equivalent resistance is

$$6 \Omega \parallel 3 \Omega = \frac{6 \times 3}{6 + 3} = 2 \Omega$$

Also, the 1 ohm and 5ohms resistors are in series; hence their equivalent resistance is

$$1 \Omega + 5 \Omega = 6 \Omega$$

Thus the circuit in Fig.(b) is reduced to that in Fig. (c). In Fig. (b), we notice that the two 2 ohms resistors are in series, so the equivalent resistance is

$$2 \Omega + 2 \Omega = 4 \Omega$$

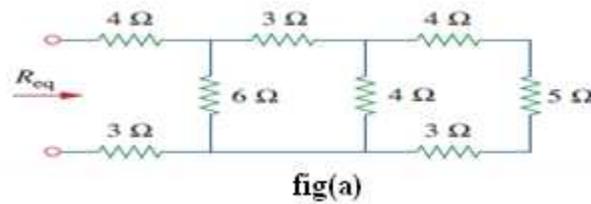
This 4 ohms resistor is now in parallel with the 6 ohms resistor in Fig.(b); their equivalent resistance is

$$4 \Omega \parallel 6 \Omega = \frac{4 \times 6}{4 + 6} = 2.4 \Omega$$

The circuit in Fig.(b) is now replaced with that in Fig.(c). In Fig.(c), the three resistors are in series. Hence, the equivalent resistance for the circuit is

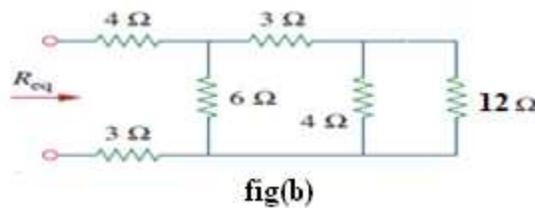
$$R_{eq} = 4 \Omega + 2.4 \Omega + 8 \Omega = 14.4 \Omega$$

2) Find the Req for the circuit shown in below figure.

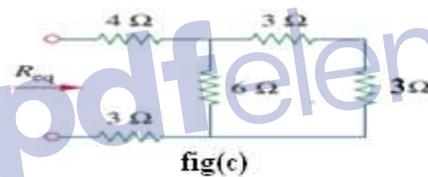


Solution:

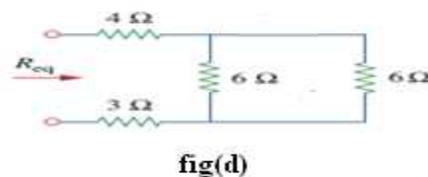
In the given network 4 ohms, 5 ohms and 3 ohms comes in series then equivalent resistance is $4+5+3=12$ ohms



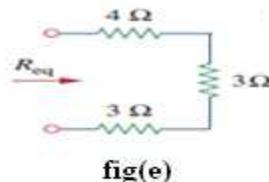
From fig(b), 4 ohms and 12 ohms are in parallel, equivalent is 3 ohms



From fig(c), 3 ohms and 3 ohms are in series, equivalent resistance is 6 ohms



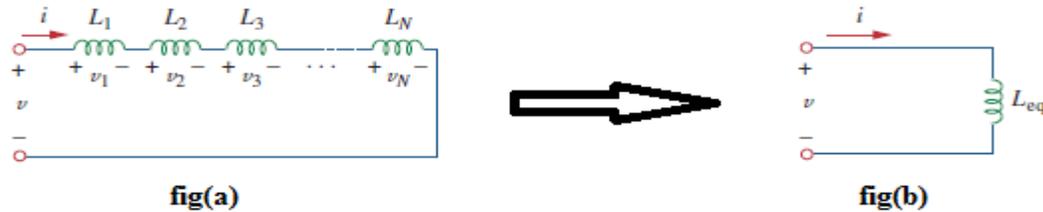
From fig(d), 6 ohms and 6 ohms are in parallel, equivalent resistance is 3 ohms



From fig(e), 4 ohms, 3 ohms and 3 ohms are in series. Hence $R_{eq} = 4+3+3=10$ ohms

Series Connection of Inductors:

Two or more inductors in a circuit are said to be in series when the current flowing through all the inductors is the same.



Consider a series connection of N inductors, as shown in Fig(a), with the equivalent circuit shown in Fig(b). The inductors have the same current through them. Applying KVL to the loop,

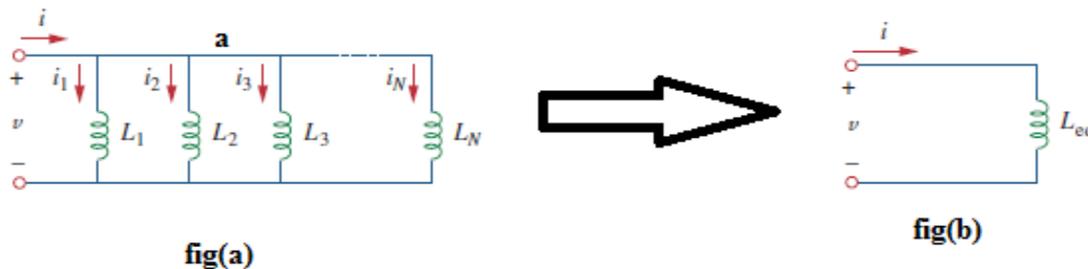
$$\begin{aligned}
 V &= V_1 + V_2 + \dots + V_n = L_1 \cdot di/dt + L_2 \cdot di/dt + \dots + L_n \cdot di/dt \\
 &= di/dt (L_1 + L_2 + \dots + L_n) \\
 &= di/dt (L_{eq})
 \end{aligned}$$

where L_{eq} is the equivalent Inductance of all the Inductances L_1, L_2, \dots and L_n in series. Hence L_{eq} of a series circuit consisting of n Inductances L_1, L_2, \dots, L_n connected in series is given by

$$L_{eq} = L_1 + L_2 + \dots + L_n$$

Note: The equivalent Inductance L_{eq} of a circuit with n Inductances L_1, L_2, \dots, L_n connected in series is equal to the sum of the individual Inductances L_1, L_2, \dots, L_n

Parallel Connection of Inductors:



Two or more inductors in a circuit are said to be in Parallel when all the circuit inductors are connected to the same nodes and the same voltage is appearing across all these inductors. Consider the circuit in fig(a) and using the Kirchhoff's Current Law (KCL) at the node 'a' the governing equation can be written as:

$$i = i_1 + i_2 + \dots + i_n \dots \dots (1)$$

In terms of the applied voltage V , the individual Inductances L_1, L_2, \dots, L_n the above equation can be written as:

$$i = (1/L_1) \int v dt + (1/L_2) \int v dt + \dots + (1/L_n) \int v dt = [(1/L_1) + (1/L_2) + \dots + (1/L_n)] \int v dt \dots (2)$$

Similarly for the circuit in fig(b) we can write the governing equation as:

$$i = (1/L_{eq}) \int v \dots (3)$$

where L_{eq} is the equivalent Inductance of all the Inductances L_1, L_2, \dots and L_n in parallel.

Since current is the same in the above two equations we find that

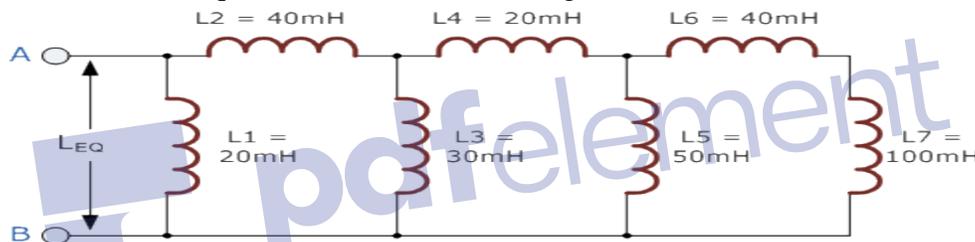
$$1/L_{eq} = (1/L_1 + 1/L_2 + \dots + 1/L_n)$$

Hence L_{eq} of a parallel circuit consisting of n Inductances L_1, L_2, \dots and L_n connected in parallel is given by :

$$1/L_{eq} = (1/L_1 + 1/L_2 + \dots + 1/L_n)$$

Note: The reciprocal of the equivalent inductance is the sum of the reciprocals of the inductances.

Problem: Determine the equivalent inductance in the given network.



Solution:

Calculating the first inductor branch L_A , where an Inductor L_5 in parallel with inductors L_6 and L_7 .

$$L_A = \frac{L_5 \times (L_6 + L_7)}{L_5 + L_6 + L_7} = \frac{50\text{mH} \times (40\text{mH} + 100\text{mH})}{50\text{mH} + 40\text{mH} + 100\text{mH}} = 36.8\text{mH}$$

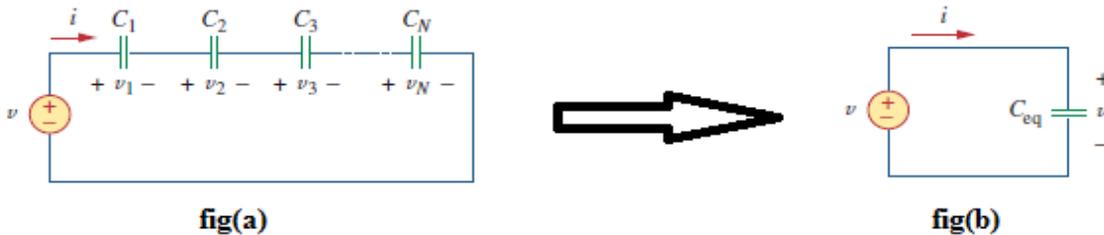
Calculating the second inductor branch L_B , where Inductor L_3 in parallel with inductors L_4 and L_A .

$$L_B = \frac{L_3 \times (L_4 + L_A)}{L_3 + L_4 + L_A} = \frac{30\text{mH} \times (20\text{mH} + 36.8\text{mH})}{30\text{mH} + 20\text{mH} + 36.8\text{mH}} = 19.6\text{mH}$$

Calculate the equivalent circuit inductance L_{EQ} , where Inductor L_1 in parallel with inductors L_2 and L_B .

$$LEQ = \frac{L1 \times (L2 + LB)}{L1 + L2 + LB} = \frac{20mH \times (40mH + 19.6mH)}{20mH + 40mH + 19.6mH} = 15mH$$

Series Connection of Capacitors:



Two or more capacitors in a circuit are said to be in series when the current flowing through all the capacitors is the same.

Fig (a) above shows n capacitances C1 ,C2, ... Cn connected in series along with a Voltage source V and V1 , V2 Vn are the voltage drops across the capacitances C1 , C2 ,....and Cn. Fig (b) shows the same circuit with it's equivalent Capacitance Ceq.

Applying KVL to the circuit in figure (a) we can write: $v = v1 + v2 + v3 + \dots + vn \dots \dots \dots (1)$

$$V = (1/C1) \int idt + (1/C2) \int idt + \dots + (1/Cn) \int idt = [(1/C1) + (1/C2) + \dots + (1/Cn)] \int idt \dots \dots \dots (2)$$

Similarly for the circuit in fig.(b) we can write the governing equation as:

$$v = (1/Ceq) \int idt \dots \dots \dots (3)$$

From the above two equations i.e eq(2) and eq (3) for 'v' we get

$$1/Ceq = (1/C1) + (1/C2) + \dots + (1/Cn)$$

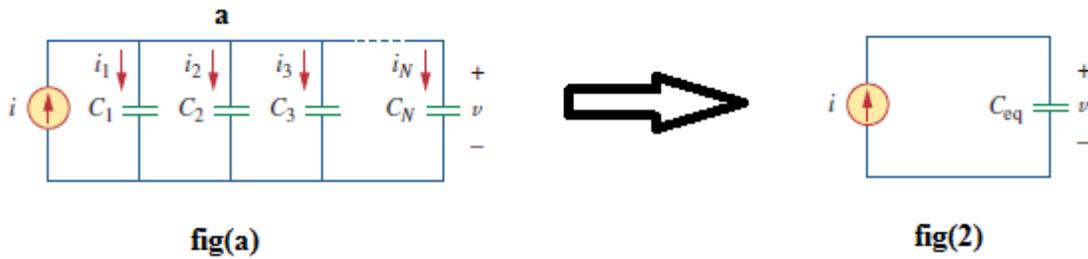
Hence Ceq of a circuit consisting of n capacitances C1 , CL2 ,....and Cn connected in series is given by :

$$1/Ceq = (1/C1) + (1/C2) + \dots + (1/Cn)$$

Note: The reciprocal of the equivalent capacitance is the sum of the reciprocals of the inductances.

Parallel Connection of Capacitors:

Two or more capacitors in a circuit are said to be in Parallel when all the capacitors are connected to the same nodes and the same voltage is appearing across all these capacitors



Applying KCL to the circuit in figure (a) we can write: $i = i_1 + i_2 + i_3 + \dots + i_n \dots (1)$

$$i = C_1 (dv/dt) + C_2 (dv/dt) + \dots + C_n dv/dt = (C_1 + C_2 + \dots + C_n) (dv/dt) \dots (2)$$

Similarly for the circuit in figure (b) we can write the governing equation as:

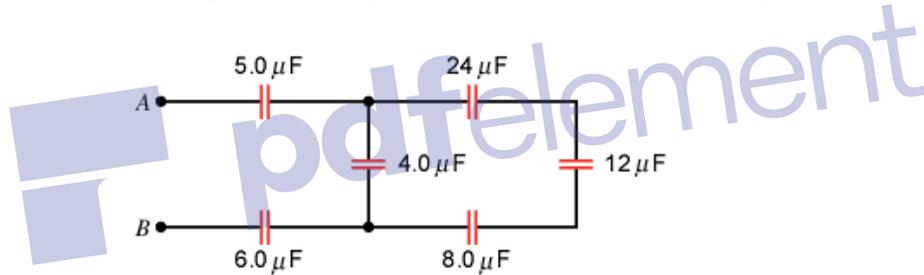
$$i = C_{eq} (dv/dt) \dots (3)$$

From the eq(2) and eq(3), we get

$$C_{eq} = C_1 + C_2 + \dots + C_n$$

Note: The equivalent capacitance of a circuit consisting of n capacitances C_1, C_2, \dots and C_n connected in parallel is the sum of the individual capacitances.

Problem: Determine the equivalent capacitance between AB in the given circuit.



Solution: In the given circuit 24uF, 12uF and 8uF are in series. The equivalent capacitance of this three capacitors is

$$\frac{1}{C_s} = \frac{1}{24 \mu F} + \frac{1}{12 \mu F} + \frac{1}{8.0 \mu F} \quad \text{or} \quad C_s = 4.0 \mu F$$

This 4.0μF capacitance is in parallel with the 4.0μF capacitance. Then the equivalent capacitance is

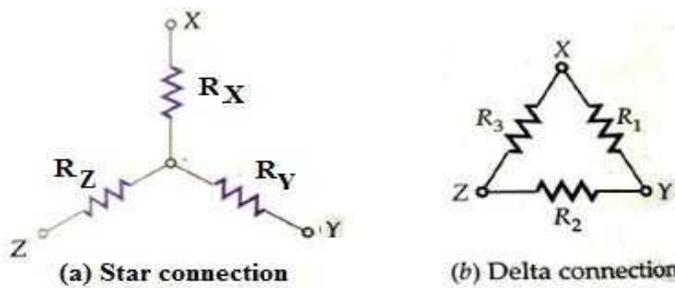
$$C_p = 4.0 \mu F + 4.0 \mu F = 8.0 \mu F$$

This 8.0μF, 5.0uF and 6.0uF are in series .Hence , the overall equivalent capacitance is

$$\frac{1}{C_s} = \frac{1}{5.0 \mu\text{F}} + \frac{1}{8.0 \mu\text{F}} + \frac{1}{6.0 \mu\text{F}} \quad \text{or} \quad C_s = \boxed{2.0 \mu\text{F}}$$

Star to Delta and Delta to Star Transformations:

Like in series and parallel connections, electrical components may be connected in Star or Delta configurations as shown in the figure below (with Resistances). Many a times circuits have to be transformed from Star to equivalent Delta and Delta to equivalent Star configurations such that the net terminal Resistances (Impedances) across the terminals are the same. We will show this transformation methodology and the resulting configurations for both Delta to Star and Star to Delta one by one.



Delta to Star Transformation:

The circuit configurations are identical provided the net resistances across the terminal pairs XY, YZ and ZX in both connections are the same. In Star Connection they are:

$$R_{X-Y} = R_X + R_Y \dots\dots\dots(1)$$

$$R_{Y-Z} = R_Y + R_Z \dots\dots\dots(2)$$

$$R_{Z-X} = R_Z + R_X \dots\dots\dots(3)$$

Similarly in Delta connection they are:

$$R_{X-Y} = R_1 // (R_2 + R_3) = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \dots\dots\dots(4)$$

$$R_{Y-Z} = R_2 // (R_1 + R_3) = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} \dots\dots\dots(5)$$

$$R_{Z-X} = R_3 // (R_1 + R_2) = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \dots\dots\dots(6)$$

By equating the respective equations, we get

$$R_X + R_Y = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \dots\dots\dots(7)$$

$$R_Y + R_Z = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} \dots\dots\dots(8)$$

$$R_Z + R_X = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \dots\dots\dots(9)$$

By subtracting equation 8 from equation 7 given above, we get

$$R_X - R_Z = \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3} - \frac{R_2R_1 + R_2R_3}{R_1 + R_2 + R_3} \dots\dots\dots(10)$$

Then adding this equation to equation 9 above i.e. (R_Z+R_X) we get:

$$2R_X = \frac{R_1R_2 + R_1R_3 - R_2R_1 - R_2R_3 + R_3R_1 + R_3R_2}{R_1 + R_2 + R_3}$$

$$R_X = \frac{R_1R_3}{R_1 + R_2 + R_3}$$

And in a similar way we can get:

$$R_Y = \frac{R_1R_2}{R_1 + R_2 + R_3}$$

$$R_Z = \frac{R_2R_3}{R_1 + R_2 + R_3}$$

Where R_X, R_Y and R_Z are the equivalent resistances in the Star connection corresponding to the Delta connection with resistances R₁, R₂ and R₃.

Star to Delta Transformation:

Now we have to get the equivalent values of R₁, R₂ and R₃ in Delta connection in terms of the three resistances R_X, R_Y and R_Z in Star connection.

Let us use the equations we got earlier i.e. R_X, R_Y and R_Z in terms of R₁, R₂ and R₃ and get the sum of the three product pairs i.e. R_XR_Y + R_YR_Z + R_ZR_X as :

$$RXRY + RYRZ + RZR X = \frac{R1^2R2R3 + R2^2R1R3 + R3^2R1R2}{(R1 + R2 + R3)^2}$$

Now let us divide this equation by R_X to get:

$$\begin{aligned} RY + RZ + \frac{R Y R Z}{R X} &= \frac{R1R2R3(R1 + R2 + R3)}{R X (R1 + R2 + R3)^2} \\ &= \frac{R1R2R3}{R X (R1 + R2 + R3)} \end{aligned}$$

Now substituting the value of $R_X = (R1+R2+R3) / R1.R3$ from the earlier equations into the above equation we get:

$$R Y + R Z + \frac{R Y R Z}{R X} = \frac{R1R2R3}{(R1 + R2 + R3)} \times \frac{(R1 + R2 + R3)}{R1R3} = R2$$

Then similarly dividing the same equation by R_Y and R_Z we get the other two relations as:

$$\begin{aligned} R X + R Z + \frac{R X R Z}{R Y} &= R3 \\ R Y + R X + \frac{R X R Y}{R Z} &= R1 \end{aligned}$$

Thus we get the three equivalent resistances $R1$, $R2$ and $R3$ in Delta connection in terms of the three resistances R_X , R_Y and R_Z in Star connection as :

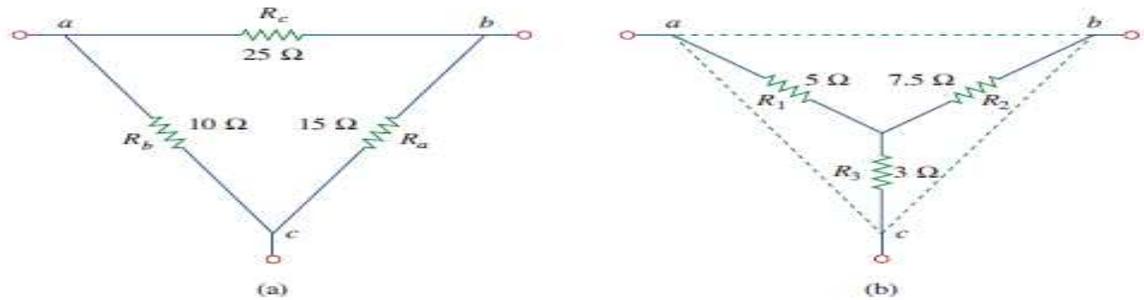
$$R Y + R X + \frac{R X R Y}{R Z} = R1$$

$$R Y + R Z + \frac{R Y R Z}{R X} = R2$$

$$R X + R Z + \frac{R X R Z}{R Y} = R3$$

Example problems:

1) Convert the Delta network in a) Fig.(a) to an equivalent star network



Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5 \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5 \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3 \Omega$$

2) Convert the star network in fig(a) to delta network



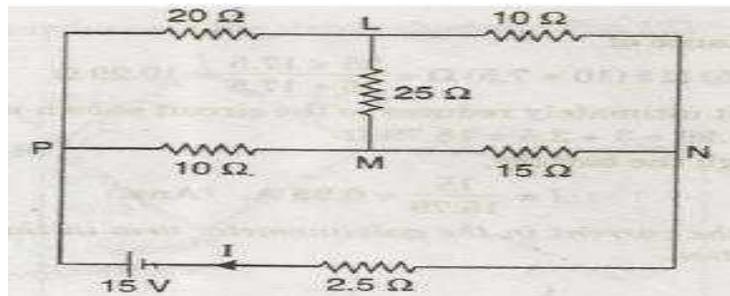
Solution: The equivalent delta for the given star is shown in fig(b), where

$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = 10 \Omega$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = 15 \Omega$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833 = 5 \Omega$$

3) Determine the total current I in the given circuit.

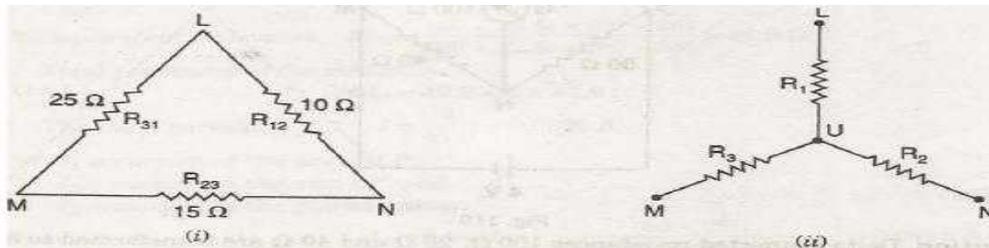


Solution: Delta connected resistors 25 ohms, 10 ohms and 15 ohms are converted in to star as shown in given figure.

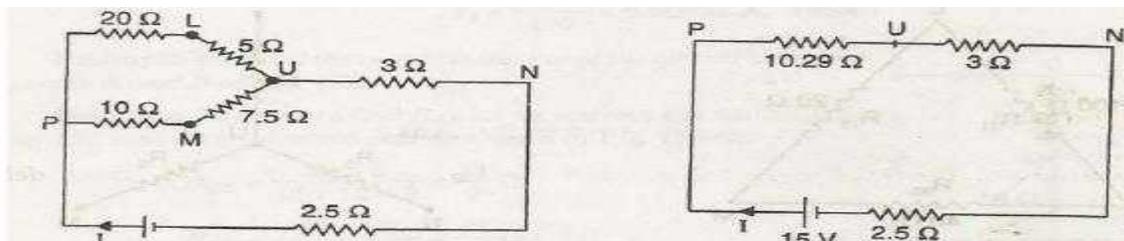
$$R_1 = R_{12} R_{31} / R_{12} + R_{23} + R_{31} = 10 \times 25 / 10 + 15 + 25 = 5 \text{ ohms}$$

$$R_2 = R_{23} R_{12} / R_{12} + R_{23} + R_{31} = 15 \times 10 / 10 + 15 + 25 = 3 \text{ ohms}$$

$$R_3 = R_{31} R_{23} / R_{12} + R_{23} + R_{31} = 25 \times 15 / 10 + 15 + 25 = 7.5 \text{ ohms}$$



The given circuit thus reduces to the circuit shown in below fig.



The equivalent resistance of

$$(20 + 5) \text{ ohms} \parallel (10 + 7.5) \text{ ohms} = 25 \times 17.5 / 25 + 17.5 = 10.29 \text{ ohms}$$

$$\text{Total resistance} = 10.29 + 3 + 2.5 = 15.79 \text{ ohms}$$

Hence the total current through the battery,

$$I = 15 / 15.79 = 0.95 \text{ A}$$

Introduction to Mesh Analysis and Nodal Analysis:

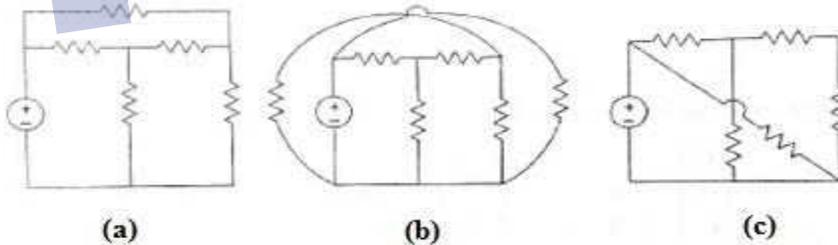
Mesh Analysis and Nodal Analysis are two important techniques used in network analysis to find out different branch currents and Node voltages. The suitability of each analysis depends mainly on the number of voltage/current sources in the given network. If the voltage sources are more Mesh analysis is suitable and if current sources are more Nodal analysis is more suitable.

Mesh Analysis:

Mesh analysis provides general procedure for analyzing circuits using mesh currents as the circuit variables. Mesh Analysis is applicable only for planar networks. It is preferably useful for the circuits that have many loops. This analysis is done by using KVL and Ohm's law.

Planar circuit: A planar circuit is one that can be drawn in a plane with no branches crossing one another. In the figure below (a) is a planar circuit.

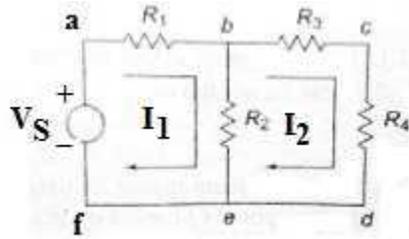
Non-Planar circuit: A non-planar circuit is one that cannot be drawn in a plane without the branches crossing one another. In the figure below (b) is a non-planar circuit and (c) is a planar circuit but appears like a non-planar circuit.



:

Loop: It is a closed path along the circuit elements.

Mesh: Mesh is a loop which does not contain any loop within it.

Mesh analysis with example:**Determination of mesh currents:****Solution:**

Step (1): Identify the no. of meshes in the given circuit.

There are two meshes..Mesh (1).....**abef** and

Mesh (2).....**bcde**

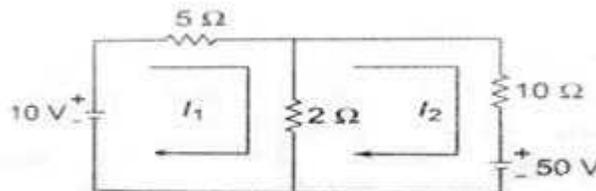
Step (2): Apply the KVL to the all meshes.

For mesh (1) by applying KVL... $V_S - I_1 \cdot R_1 + (I_1 - I_2) \cdot R_2 = 0$(1)

For mesh (2) by applying KVL.... $I_2 \cdot (R_3) + I_2 (R_4) + (I_2 - I_1) \cdot R_2 = 0$(2)

Step (3): solve the above equations for mesh currents.

Problem: Write down the mesh current equations for the circuit shown in the figure below and determine the currents I_1 and I_2 .

**Solution:**

By applying KVL to the two meshes, we get

$$5 I_1 + 2(I_1 - I_2) = 10$$

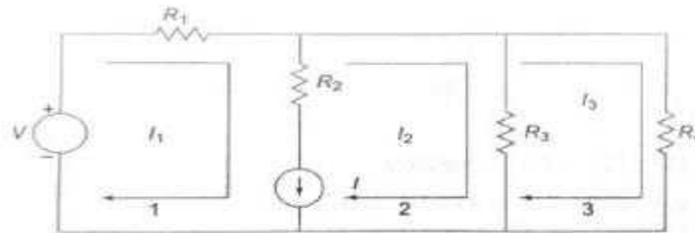
$$10 I_2 + 2(I_2 - I_1) = -50.$$

Solving the above equations gives.... $I_1 = 0.25$ A and $I_2 = -4.25$ A. The negative sign for the current I_2 indicates that it flows in the opposite direction to that assumed in the loop two.

Super Mesh Analysis: If there is only current source between two meshes in the given network then it is difficult to apply the mesh analysis. Because the current source has to be converted into a voltage source in terms of the current source, write down the mesh equations and relate the mesh currents to the current source. But this is a difficult approach. This difficulty can be avoided by creating super mesh which encloses the two meshes that have common current source

Super Mesh: A super mesh is constituted by two adjacent meshes that have a common current source.

Let us illustrate this method with the following simple generalized circuit.



Solution:

Step (1): Identify the position of current source.

Here the current source is common to the two meshes 1 and 2. so, super mesh is nothing but the combination of meshes 1 and 2.

Step (2): Apply KVL to super mesh and to other meshes

Applying KVL to this super mesh (combination of meshes 1 and 2) we get

$$R_1 I_1 + R_3 (I_2 - I_3) = V \dots \dots \dots (1)$$

Applying KVL to mesh 3, we get

$$R_3 (I_3 - I_2) + R_4 I_3 = 0 \dots \dots \dots (2)$$

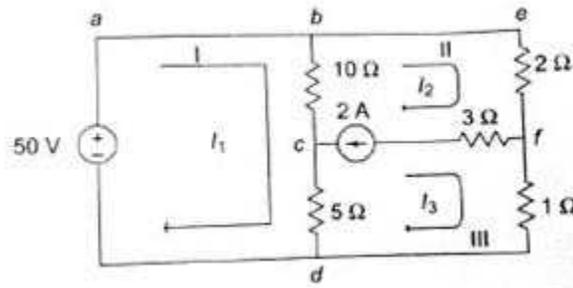
Step (3): Make the relation between mesh currents with current source to get third equation.

Third equation is nothing but the relation between I , I_1 and I_2 which is

$$I_1 - I_2 = I \dots \dots \dots (3)$$

Step(4): Solve the above equations to get the mesh currents.

Example(1): Determine the current in the 5 Ω resistor shown in the figure below.



Solution:

Step(1): Here the current source exists between mesh(2) and mesh(3). Hence, super mesh is the combination of mesh(2) and mesh(3). Applying KVL to the super mesh (combination of mesh 2 and mesh 3 after removing the branch with the current source of 2 A and resistance of 3 Ω) we get :

$$10(I_2 - I_1) + 2I_2 + I_3 + 5(I_3 - I_1) = 0$$

$$-15I_1 + 12I_2 + 6I_3 = 0 \dots\dots\dots(1)$$

Step (2): Applying KVL first to the normal mesh 1 we get :

$$10(I_1 - I_2) + 5(I_1 - I_3) = 50$$

$$15I_1 - 10I_2 - 5I_3 = 50 \dots\dots\dots(2)$$

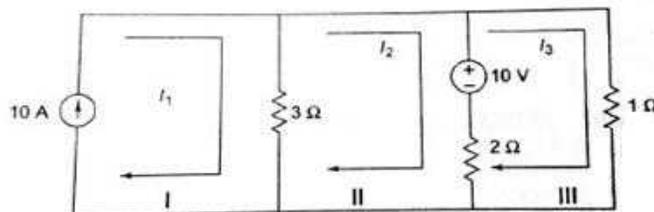
Step (3): We can get the third equation from the relation between the current source of 2 A , and currents I₂ & I₃ as :

$$I_2 - I_3 = 2 \text{ A} \dots\dots\dots(3)$$

Step (4): Solving the above three equations for I₁, I₂ and I₃ we get I₁ = 19.99 A I₂ = 17.33 A and I₃ = 15.33 A

The current in the 5 Ω resistance = I₁ - I₃ = 19.99 - 15.33 = 4.66 A

Example(2): Write down the mesh equations for the circuit shown in the figure below and find out the values of the currents I₁, I₂ and I₃



Solution: In this circuit the current source is in the perimeter of the circuit and hence the first mesh is ignored. So, here no need to create the super mesh.

Applying KVL to mesh 1 we get :

$$3(I_2 - I_1) + 2(I_2 - I_3) = -10$$

$$-3.I_1 + 5.I_2 - 2.I_3 = -10.....(1)$$

Next applying KVL to mesh 2 we get :

$$I_3 + 2(I_3 - I_2) = 10$$

$$-2.I_2 + 3.I_3 = 10.....(2)$$

And from the first mesh we observe that..... $I_1 = 10$ A.....(3)

And solving these three equations we get : $I_1 = 10$ A, $I_2 = 7.27$ A, $I_3 = 8.18$ A

Nodal analysis:

Nodal analysis provides another general procedure for analyzing circuits nodal voltages as the circuit variables. It is preferably useful for the circuits that have many no. of nodes. It is applicable for the both planar and non planar circuits. This analysis is done by using KCL and Ohm's law.

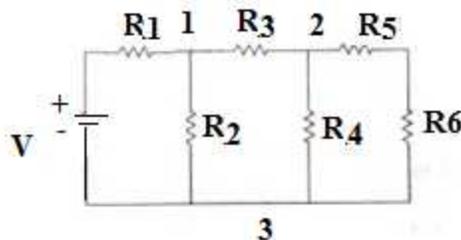
Node: It is a junction at which two or more branches are interconnected.

Simple Node: Node at which only two branches are interconnected.

Principal Node: Node at which more than two branches are interconnected.

Nodal analysis with example:

Determination of node voltages:

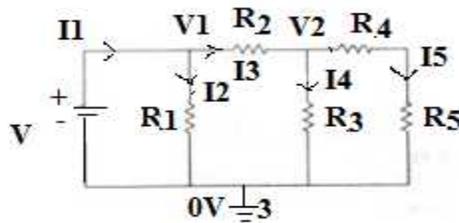


Procedure:

Step (1): Identify the no. nodes, simple nodes and principal nodes in the given circuit. Among all the nodes one node is taken as reference node. Generally bottom is taken as reference node. The potential at the reference node is 0v.

In the given circuit there are 3 principal nodes in which node (3) is the reference node.

Step (2): Assign node voltages to the all the principal nodes except reference node and assign branch currents to all branches.



Step (3): Apply KCL to those principal nodes for nodal equations and by using ohm's law express the node voltages in terms of branch current.

Applying KCL to node (1)---- $I_1 = I_2 + I_3$

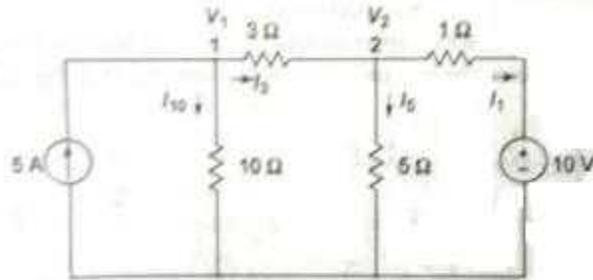
Using ohm's law, we get $(V - V_1)/R_1 = (V_2 - 0)/R_2 + (V_1 - V_2)/R_3$(1)

Applying KCL to node (2)---- $I_3 = I_4 + I_5$

Using ohm's law, we get $(V_1 - V_2)/R_3 = (V_2 - 0)/R_4 + (V_2 - 0)/R_5$ (2)

Step(4): Solve the above nodal equations to get the node voltages.

Example: Write the node voltage equations and find out the currents in each branch of the circuit shown in the figure below.



Solution:

The node voltages and the directions of the branch currents are assigned as shown in given figure.

Applying KCL to node 1, we get: $5 = I_{10} + I_3$

$$5 = (V_1 - 0)/10 + (V_1 - V_2)/3$$

$$V_1(13/30) - V_2(1/3) = 5 \dots\dots\dots(1)$$

Applying KCL to node 2, we get: $I_3 = I_5 + I_1$

$$(V_1 - V_2)/3 = (V_2 - 0)/5 + (V_2 - 10)/1$$

$$V_1(1/3) - V_2(23/15) = -10 \dots\dots\dots(2)$$

Solving the these two equations for V1 and V2 we get :

V1 = 19.85 V and V2 = 10.9 V and the currents are :

$$I_{10} = V_1/10 = 1.985A$$

$$I_3 = (V_1 - V_2)/3 = (19.85 - 10.9)/3 = 2.98A$$

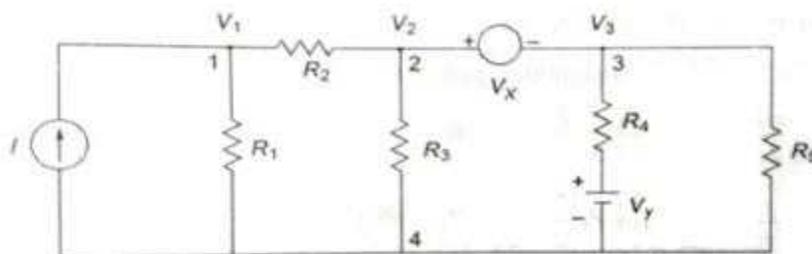
$$I_5 = V_2/5 = 10.9/5 = 2.18A$$

$$I_1 = (V_2 - 10) = (10.9 - 10)/1 = 0.9A$$

Super Node Analysis: If there is only voltage source between two nodes in the given network then it is difficult to apply the nodal analysis. Because the voltage source has to be converted into a current source in terms of the voltage source, write down the nodal equations and relate the node voltages to the voltage source. But this is a difficult approach .This difficulty can be avoided by creating super node which encloses the two nodes that have common voltage source.

Super Node: A super node is constituted by two adjacent nodes that have a common voltage source.

Example: Write the nodal equations by using super node analysis.



Procedure:

Step(1):Identify the position of voltage source.Here the voltage source is common to the two nodes 2 and 3.so, super node is nothing but the combination of nodes 2 and 3 .

Step (2):Apply KCL to super node and to other nodes.

Applying KCL to this super node (combination of meshes 2 and 3), we get

$$(V_2-V_1)/R_2 + V_2/R_3 + (V_3-V_1)/R_4 + V_3/R_5 = 0.....(1)$$

Applying KVL to node 1 ,we get

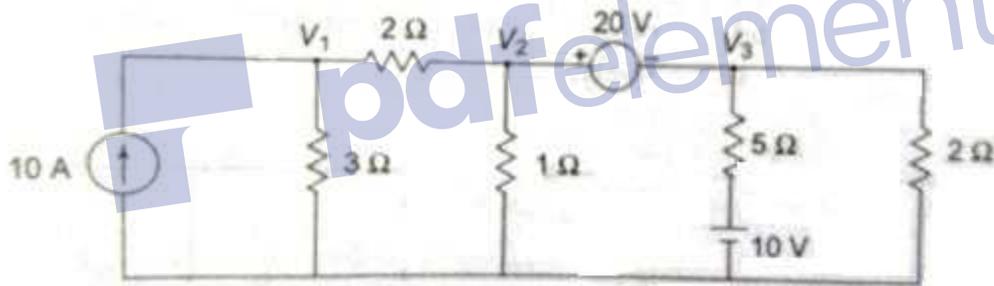
$$I = V_1/R_1 + (V_1-V_2)/R_2.....(2)$$

Step (3): Make the relation between node voltages with voltage source to get third equation.

Third equation is nothing but the relation between V_X , V_2 and V_3 which is $V_2 - V_3 = V_x.....(3)$

Step (4): Solve the above nodal equations to get the node voltages.

Example: Determine the current in the 5 Ω resistor shown in the circuit below



Solution:

Applying KCL to node 1: $10 = V_1/3 + (V_1-V_2)/2$

$$V_1 [1/3 + 1/2] - V_2 /2 = 10$$

$$0.83 V_1 - 0.5 V_2 = 10.....(1)$$

Next applying KCL to the super node2&3 :

$$(V_2-V_1)/2 + V_2/1 + (V_3-10)/5 + V_3/2 = 0$$

$$-V_1/2 + V_2(1/2 + 1) + V_3(1/5 + 1/2) = 10$$

$$0.5 V_1 + 1.5V_2 + 0.7 V_3 = 2.....(2)$$

and the third and final equation is:

$$V_2 - V_3 = 20.....(3)$$

Solving the above three equations we get $V_3 = -8.42 \text{ V}$

The current through the 5Ω resistor $I_5 = [-8.42 - 10] / 5 = -3.68 \text{ A}$

The negative sign indicates that the current flows towards the node 3.



NETWORK TOPOLOGY

After a thorough study of several circuits, it slowly becomes evident that many of the circuits we see have something in common, at least in terms of the arrangement of components. From this understanding, it is possible to create a more simplified view of circuits which we call network topology.

Basic Definitions:

Topology: It is a branch of geometry which is concerned with those properties of a geometrical figure which are unchanged when the figure is twisted, bent, folded, stretched, squeezed, or tied in knots, with the provision that no parts of the figure are to be cut apart or to be joined together. A sphere and tetrahedron are topologically identical, as are a square and a circle. In terms of electric circuits, then, we are not now concerned with the particular types of elements appearing in the circuit, but only with the way in which branches and nodes are arranged. As a matter of fact, we usually suppress the nature of the elements and simplify the drawing of the circuit by showing the elements as lines. The resultant drawing is called a linear graph, or simply a graph.

A circuit and its graph are shown in Fig.1(a) and (b) below. Note that all nodes are identified by heavy dots in the graph. Since the topological properties of the circuit or its graph are unchanged when it is distorted, the three graphs shown in Fig. 2 below are all topologically identical with the circuit and graph of Fig.1

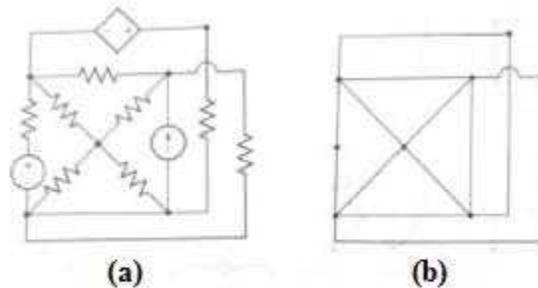


Figure 1: (a) The circuit (b) Its Graph

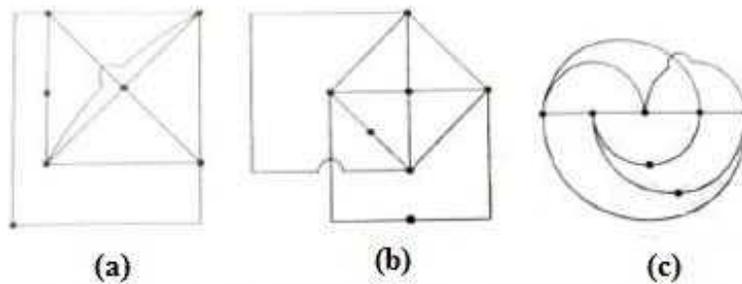


Figure 2 : The three Topologically Identical graphs of Figure 1 above.

Important terms in Topology:

Graph: When all the elements in a network like Resistors, Inductors, Capacitors etc are replaced by line segments with their end points shown as dots or circles, voltage source with short circuit and current source with open circuit is called the graph of the network.

Directed (or Oriented) graph: A graph is said to be directed (or oriented) when all the nodes and branches are numbered or direction assigned to the branches by arrow.

Node: A point at which two or more elements have a common connection.

Degree of Node: Number of branches incident on it.

Path: A set of elements that may be traversed in order without passing through the same node twice.

Branch: A single path, containing one simple element, or a combination of elements which connects one node to any other node. A simple line segment with it's two distinct end points (Nodes) represent a branch. It does not indicate anything about the nature of the element/s .

Loop: A closed path in the oriented graph is called as loop.

Mesh: A loop which does not contain any other loops within it.

Tree: It is that part of a Graph with an interconnected open set of branches which include all the nodes of the given graph. In a tree of the graph there cannot be any closed path.

Properties of a Tree:

- (i) It consists of all the nodes of the graph.
- (ii) If the graph has N nodes, then the tree has $(N-1)$ branch.
- (iii) There will be no closed path in a tree
- (iv) There can be many possible different trees for a given graph depending on the no. of nodes and branches.

Cotree: Those branches that are not part of the tree form the cotree, or complement of the tree. The lightly drawn branches in **Fig.3 b to e** show the cotrees that correspond to the heavier trees.

Twig (Tree branch): All branches of a tree are called Twigs.

Link (Chord): It is that branch of a graph that does not form part of the tree and when included makes the tree or a part of it a loop. In other words it is simply any branch belonging to the cotree.

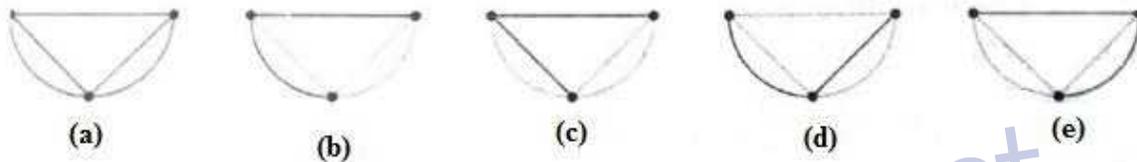
Planar circuit: A circuit which may be drawn on a plane surface in such a way that no branch passes above or beneath any other branch.

Non-planar circuit: Any circuit which is not planar.

The figure below shows a simple three node Graph in figure (a) and four of the eight possible trees that can be drawn. Note that

- Nodes are drawn as dots
- Twigs are shown dark
- Links are shown dotted

Figure 3: (a) The Graph of a three Node network. (b,c,d,e) Four of the eight different Possible Trees



Relation between nodes, branches, twigs and links:

Let N = no. of nodes

L = total no. of links

B = total no. of branches

No. of twigs = $N - 1$

Then, $L = B - (N - 1)$

or

$$L = B - N + 1$$

There are L branches in the cotree and $(N - 1)$ branches in the tree.

Incidence Matrix (A):

Any oriented graph can be described completely in a compact matrix form. Here we specify the orientation of each branch in the graph and the nodes at which this branch is incident. This branch is called incident matrix. When one row is completely deleted from the matrix the remaining matrix is called a reduced incidence matrix. Order of incidence matrix is $(n \times b)$.

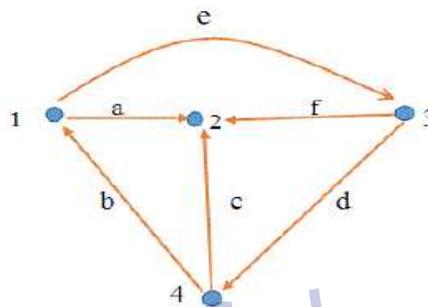
Properties of incidence matrix:

1. Number of non zero entries of row indicates degree of the node.
2. The non zero entries of the column represents branch connections.
3. If two columns has same entries then they are in parallel.

Procedure to form incidence matrix:

- $a_{ij} = 1$, if j th branch is incidence to i th node and direction is away from node.
- $a_{ij} = -1$, if j th branch is incidence to i th node and direction is towards from node.
- $a_{ij} = 0$, if j th branch is not incidence to i th node .

Example: Draw incidence matrix for the given graph.



Solution: In the above shown graph or directed graph, there are 4 nodes and 6 branches. Thus the incidence matrix for the above graph will have 4 rows and 6 columns. For the graph shown above write its incidence matrix.

$[A_C] =$

nodes \ branches	a	b	c	d	e	f
1	1	-1	0	0	1	0
2	-1	0	-1	0	0	-1
3	0	0	0	1	-1	1
4	0	1	1	-1	0	0

Reduced Incidence Matrix:

If from a given incidence matrix $[A_C]$, any arbitrary row is deleted, then the new matrix formed will be reduced incidence matrix. It is represented by symbol $[A]$. The order of reduced incidence matrix is $(n-1)*b$ where n is the number of nodes and b is the number of branches.

$[A] =$

nodes \ branches	a	b	c	d	e	f
1	1	-1	0	0	1	0
2	-1	0	-1	0	0	-1
3	0	0	0	1	-1	1

[NOTE: In the above shown matrix row 4 is deleted.]

Tie-set: It is a unique set with respect to a given tree of a connected graph containing one chord and all of the free branches contained in the free path formed between two vertices of the chord.

Tie-set Matrix (Loop matrix):

This matrix is used to find the branch currents. For a given tree of a graph addition of each link forms a closed path and in that closed path current flows which is also the link current. The current in any branch of a graph can be found by using the link currents and their direction.

Fundamental tie-set Matrix (Fundamental loop matrix):

A fundamental loop or a fundamental tie set of a graph with respect to a tree is a loop formed by only one link associated with other twigs.

Since for each link of the tree there will be a corresponding fundamental loop, the number of fundamental loops is equal to the number of links in that tree. i.e.

$$\text{Number of fundamental loops} = B - (N-1)$$

Procedure for forming the fundamental tie-set Matrix:

1. A tree is selected arbitrarily in the graph.
2. Fundamental loops are formed with each link in the graph for the entire tree.
3. Directions of the loop currents are oriented in the same direction as that of the concerned link.
4. Fundamental tie-set matrix $[B_{ij}]$ is formed where
 - $B_{ij} = 1$ when branch b_j is in the fundamental loop i and their reference directions are same
 - $B_{ij} = -1$ when branch b_j is in the fundamental loop i and their reference directions are opposite.
 - $B_{ij} = 0$ when branch b_j is not in the fundamental loop i .

Illustration:

An oriented graph is shown in the figure 1(a) below. Lets us select a tree arbitrarily as shown in the figure 1(b) below. Then, the loops (tie sets) are formed as shown in the figure 1(c) below

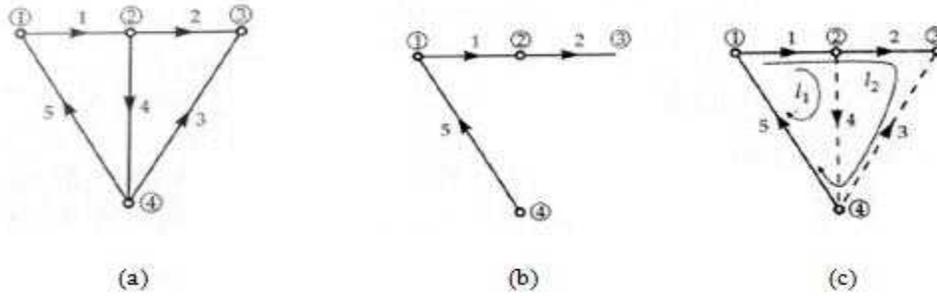


Figure 3: (a) An oriented Graph (b) One of its Tree (c) The Loops (tie sets) of the Tree

There are only two fundamental loops as there are only two links as explained below:

1. Loop-1 : Has current I_1 and is formed with Twigs-1&5 and Link-4
2. Loop-2 : Has current I_2 and is formed with Twigs-1,2&5 and Link-3

Now let us find out the values of the elements of the tie-set matrix $[[B_{ij}]$ applying the rule at step 4 in the procedure given above:

1. Loop-1:

- a. Elements Q_{11} , Q_{14} and Q_{15} are zero since the branches 1,4 and 5 are not linked with cut-set-1
- b. Elements Q_{12} and Q_{13} are +1 since the twig 2 and branch 3 are linked with cut-set-1 and also in the same orientation as that of cut-set-1.

2. Loop-2 :

- a. Elements B_{11} , B_{14} and B_{15} are +1 since the branches 1 and 2 are not linked with cut-set-2
- b. Elements Q_{23} and Q_{25} are +1 since the branches 3 and 5 are linked with cut-set-2 and are also in the same orientation as that of cut-set-2.
- c. Elements Q_{24} is -1 since the branch 4 is linked with cut-set-2 but its orientation is opposite to that of cut-set-2.

Tie-set Matrix:

Branches	1	2	3	4	5
Loops or Tie-sets					
Loop(1)	1	0	0	1	1
Loop(2)	1	1	-1		1

Cut-set: It is that set of elements or branches of a graph that separates two main parts of a network. If any branch of the cut-set is not removed the network remains connected. The term cut-set is derived from the property by which the network can be divided into two parts.

A cut-set is shown on a graph by a dashed line which passes through the branches defining the cutset. A graph should have at least one cutset though there can be more than one cut-set in any graph.

Fundamental cut-set:

A fundamental cut set of a graph with respect to a tree is a cut set formed by one and only one twig and a set of links. Thus in a graph, for each twig of a chosen tree, there would be a fundamental cut-set. For a graph having N nodes there will be $(N-1)$ fundamental cut-sets (i.e. equal to the number of twigs).

As a convention, the orientation of cutset is so chosen that it coincides with the orientation of its twig.

Cut-set Matrix: This matrix provides a compact and effective means of writing all the algebraic equations giving branch voltages in terms of the tree branches.

Procedure for forming the fundamental Cut-set Matrix:

1. A tree is selected arbitrarily in the graph.
2. Fundamental cut-sets are formed (i.e. The network is divided into two parts) with each twig in the graph for the entire tree.
3. Directions of the cut-sets are oriented in the same direction as that of concerned twig.
4. Fundamental cut-set matrix $[Q_{kj}]$ is formed where

- $Q_{kj} = 1$ when branch b_j has same orientation as that of the cut-set k
- $Q_{kj} = -1$ when branch b_j has opposite orientation to that of the cut-set k
- $Q_{kj} = 0$ when branch b_j is not in the cut-set k

Illustration:

An oriented graph is shown in the figure 1(a) below. Lets us select a tree arbitrarily as shown in the figure 1(b) below. Then, cut-sets are formed as shown in the figure 1(c) below.

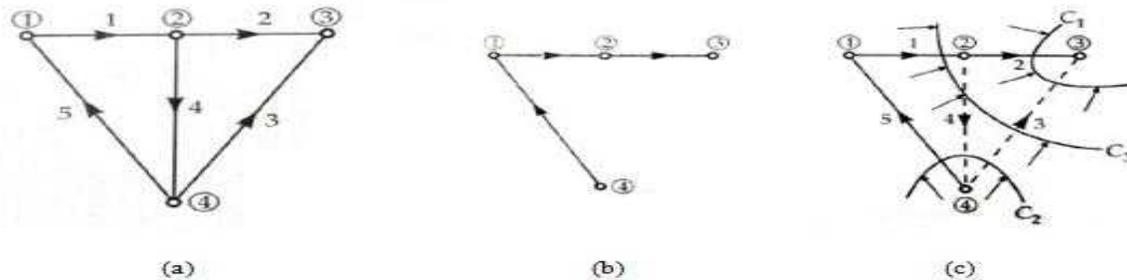


Figure 3: (a) An oriented Graph (b) One of its Tree (c) The cut-sets of the Tree

The three fundamental cut-sets are given below

1. Cut-Set-1 : Twig-2 and Link-3
2. Cut-Set-2 : Twig-5 and Links-3&4
3. Cut-Set-3 : Twig-1 and Links-4&3

The following points are to be noted:

1. The number of fundamental Cut-sets are same as the number of Twigs represented by bold lines.
2. Each Cut-set has only one twig.
3. The direction of the- cut-sets is indicated with arrows on them (same as that of the corresponding twig).

Now let us find out the values of the elements of the cut-set matrix $[[Q_{kj}]$ applying the rule at step 4 in the procedure given above:

1. Cut-Set-1 :

- a. Elements Q_{11} , Q_{14} and Q_{15} are zero since the branches 1,4 and 5 are not linked with cut-set-1
- b. Elements Q_{12} and Q_{13} are +1 since the twig 2 and branch 3 are linked with cut-set-1 and also in the same orientation as that of cut-set-1.

2. Cut-Set-2 :

- a. Elements Q_{21} and Q_{22} are zero since the branches 1 and 2 are not linked with cut-set-2
- b. Elements Q_{23} and Q_{25} are +1 since the branches 3 and 5 are linked with cut-set-2 and are also in the same orientation as that of cut-set-2.
- c. Elements Q_{24} is -1 since the branch 4 is linked with cut-set-2 but its orientation is opposite to that of cut-set-2.

3. Cut-Set-3 :

- a. Elements Q_{31} and Q_{33} are +1 since the branches 1 and 3 are linked with cut-set-3 and are also in the same orientation as that of cut-set-3.
- b. Elements Q_{32} and Q_{35} are zero since the twigs 2 and 5 are not linked with cut-set-3
- c. Elements Q_{34} is -1 since the branch 4 is linked with cut-set-3 but its orientation is opposite to that of cut-set-3.

Thus we can now frame the fundamental cut-set matrix as shown below:

Now using this matrix the current equations can be written as below:

Cut-set Matrix:

Branches	1	2	3	4	5
Cut-sets					
Cut-set(1)	0	1	1	0	0
Cut-set(2)	0	0	1	-1	1
Cut-set(3)	1	0	1	-1	0



UNIT-3:SINGLE PHASE AC CIRCUITS

- Average value, RMS Value, Form factor and Peak factor for different Periodic waveforms.
- J-Notation, Complex and Polar forms of Representation.
- Steady state Analysis of Series RLC circuits.
- Concept of Reactance, Impedance, Susceptance and Admittance.
- Phase and phase difference
- Concept of power factor, real, reactive and complex power



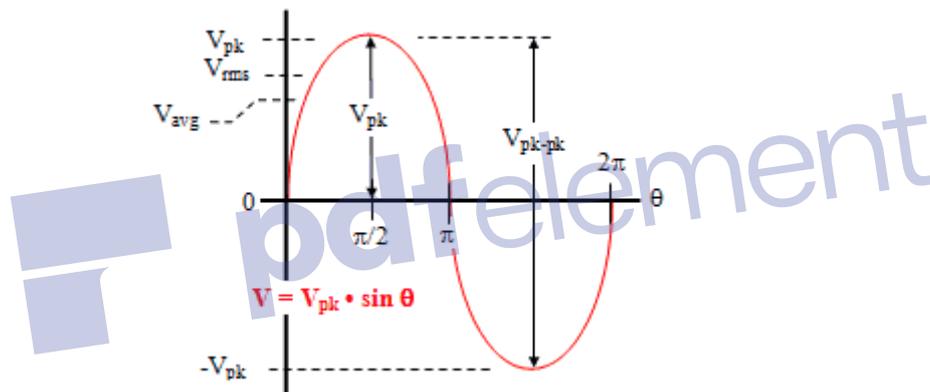
Average value, RMS Value, Form factor and Peak factor for different waveforms:

Sinusoidal wave:

A sinewave is defined by the trigonometric sine function. When plotted as voltage (V) as a function of phase (θ), it looks similar to the figure to the below. The waveform repeats every 2π radians (360°), and is symmetrical about the voltage axis (when no DC offset is present). Voltage and current exhibiting cyclic behavior is referred to as alternating; i.e., alternating current (AC). One full cycle is shown here. The basic equation for a sinewave is as follows:

$$V(\theta) = V_{pk} \cdot \sin(\theta)$$

There are a number of ways in which the amplitude of a sinewave is referenced, usually as peak voltage (V_{pk} or V_p), peak-to-peak voltage (V_{pp} or V_{p-p} or V_{pkpk} or V_{pk-pk}), average voltage (V_{av} or V_{avg}), and root-mean-square voltage (V_{rms}). Peak voltage and peak-to-peak voltage are apparent by looking at the above plot. Root-mean-square and average voltage are not so apparent.



Average Voltage (V_{avg})

As the name implies, V_{avg} is calculated by taking the average of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the sinewave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through $\pi/2$ radians (90°).

As with the V_{rms} formula, a full derivation for the V_{avg} formula is given here as well.

$$\begin{aligned} V_{avg} &= \frac{1}{\pi/2} \cdot \int_0^{\pi/2} V_{pk} \cdot \sin \theta \cdot d\theta = \frac{2}{\pi} \cdot V_{pk} \cdot -\cos \theta \Big|_0^{\pi/2} \\ &= \frac{-2}{\pi} \cdot V_{pk} \cdot \left(\cos \frac{\pi}{2} - \cos 0 \right) = \frac{-2}{\pi} \cdot V_{pk} \cdot (0 - 1) = \frac{2}{\pi} \cdot V_{pk} \end{aligned}$$

$$\text{So, } V_{\text{avg}} = \frac{2}{\pi} \cdot V_{\text{pk}} \approx 0.636 V_{\text{pk}}$$

Root-Mean-Square Voltage (V_{rms})

As the name implies, V_{rms} is calculated by taking the square root of the mean average of the square of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the sine wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through $\pi/2$ radians (90°).

V_{rms} is the value indicated by the vast majority of AC voltmeters. It is the value that, when applied across a resistance, produces that same amount of heat that a direct current (DC) voltage of the same magnitude would produce. For example, 1 V applied across a 1Ω resistor produces 1 W of heat. A 1 V_{rms} sine wave applied across a 1Ω resistor also produces 1 W of heat. That 1 V_{rms} sine wave has a peak voltage of $\sqrt{2}$ V (≈ 1.414 V), and a peak-to-peak voltage of $2\sqrt{2}$ V (≈ 2.828 V).

Since finding a full derivation of the formulas for root-mean-square (V_{rms}) voltage is difficult, it is done here for you.

$$\begin{aligned} V_{\text{rms (sinewave)}} &= \sqrt{\frac{1}{\pi/2} \int_0^{\pi/2} (V_{\text{pk}} \sin\theta)^2 \cdot d\theta} = \sqrt{\frac{2 \cdot V_{\text{pk}}^2}{\pi} \left(\frac{\theta}{2} - \frac{1}{4} \sin(2 \cdot \theta) \right) \Big|_0^{\pi/2}} \\ &= \frac{\sqrt{2} \cdot V_{\text{pk}}}{\sqrt{\pi}} \sqrt{\left(\frac{\pi/2}{2} - \frac{1}{4} \sin(\pi) \right) - \left(\frac{0}{2} - \frac{1}{4} \sin(0) \right)} \\ &= \frac{\sqrt{2} \cdot V_{\text{pk}}}{\sqrt{\pi}} \sqrt{\left(\frac{\pi}{4} - 0 \right) - (0 - 0)} = \frac{\sqrt{2} \cdot V_{\text{pk}}}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} = \frac{1}{\sqrt{2}} \cdot V_{\text{pk}} \\ \text{So, } V_{\text{rms}} &= \frac{1}{\sqrt{2}} \cdot V_{\text{pk}} \approx 0.707 V_{\text{pk}} \end{aligned}$$

Form factor:

Two alternating periodic waveforms of the same amplitude and frequency may look different depending upon their wave shape/form and then their average & RMS values will be different. In order to compare such different waveforms of the same frequency and amplitude but of different wave shape a parameter called Form factor is defined as the ratio of its RMS and Average values.

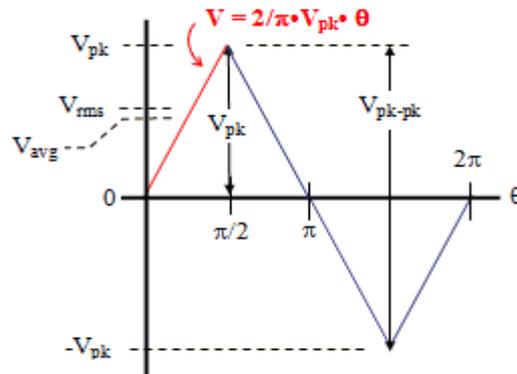
For a sinusoidal signal of peak voltage V_m it is given by :

$$\begin{aligned} \text{Form factor of a sinusoidal signal} &= V_{\text{rms}} / V_{\text{av}} \\ &= 0.707 V_m / 0.637 V_m = 1.11 \end{aligned}$$

Peak Factor (Or Crest factor): Is defined as the ratio of maximum value to the R.M.S value of an alternating quantity.

$$\begin{aligned}\text{Peak factor of a sinusoidal signal} &= V_{\max}/V_{\text{rms}} \\ &= V_{\max}/(0.707 V_m) \\ &= 1.414\end{aligned}$$

Triangular wave:



When plotted as voltage (V) as a function of phase (θ), a triangle wave looks similar to the figure to the above. The waveform repeats every 2π radians (360°), and is symmetrical about the voltage axis (when no DC offset is present). Voltage and current exhibiting cyclic behavior is referred to as alternating; i.e., alternating current (AC). One full cycle is shown here. The basic equation for a triangle wave is as follows:

$$V = \frac{2}{\pi} \cdot V_{pk} \cdot \theta \quad \text{for } 0 \leq \theta < \pi/2$$

There are a number of ways in which the amplitude of a triangle wave is referenced, usually as peak voltage (V_{pk} or V_p), peak-to-peak voltage (V_{pp} or V_{p-p} or V_{pkpk} or V_{pk-pk}), average voltage (V_{av} or V_{avg}), and root-mean-square voltage (V_{rms}). Peak voltage and peak-to-peak voltage are apparent by looking at the above plot. Root-mean-square and average voltage are not so apparent.

Average Voltage (V_{avg})

As the name implies, V_{avg} is calculated by taking the average of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the triangle wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through π/2 radians (90°).

As with the V_{rms} formula, a full derivation for the V_{avg} formula is given here as well.

$$\begin{aligned}V_{\text{avg}} &= \frac{1}{\pi/2} \int_0^{\pi/2} \frac{2}{\pi} \cdot V_{pk} \cdot \theta \cdot d\theta = \frac{2}{\pi} \cdot \frac{2}{\pi} \cdot V_{pk} \cdot \frac{1}{2} \cdot \theta^2 \Big|_0^{\pi/2} \\ &= \frac{4}{\pi^2} \cdot V_{pk} \cdot \frac{1}{2} \cdot \left(\frac{\pi^2}{4} - 0 \right) = \frac{1}{2} \cdot V_{pk}\end{aligned}$$

$$\begin{aligned}V_{\text{avg}} &= \frac{1}{2} \cdot V_{pk} \\ &\approx 0.5 V_{pk}\end{aligned}$$

Root-Mean-Square Voltage (V_{rms})

As the name implies, V_{rms} is calculated by taking the square root of the mean average of the square of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the triangle wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through $\pi/2$ radians (90°).

V_{rms} is the value indicated by the vast majority of AC voltmeters. It is the value that, when applied across a resistance, produces that same amount of heat that a direct current (DC) voltage of the same magnitude would produce. For example, 1 V applied across a 1Ω resistor produces 1 W of heat. A 1 V_{rms} triangle wave applied across a 1Ω resistor also produces 1 W of heat. That 1 V_{rms} triangle wave has a peak voltage of $\sqrt{3}$ V (≈ 1.732 V), and a peak-to-peak voltage of $2\sqrt{3}$ V (≈ 3.464 V).

Since finding a full derivation of the formulas for root-mean-square (V_{rms}) voltage is difficult, it is done here for you.

$$\begin{aligned} V_{rms} &= \sqrt{\frac{1}{\pi/2} \cdot \int_0^{\pi/2} \left(\frac{2}{\pi} \cdot V_{pk} \cdot \theta \right)^2 d\theta} = \sqrt{\frac{2}{\pi} \cdot \frac{4}{\pi^2} \cdot V_{pk}^2 \cdot \int_0^{\pi/2} \theta^2 d\theta} = \sqrt{\frac{8}{\pi^3} \cdot V_{pk}^2 \cdot \int_0^{\pi/2} \theta^2 d\theta} \\ &= \sqrt{\frac{8}{\pi^3} \cdot V_{pk}^2 \cdot \frac{1}{3} \cdot \theta^3 \Big|_0^{\pi/2}} = \sqrt{\frac{8}{3 \cdot \pi^3} \cdot V_{pk}^2 \cdot \left[\left(\frac{\pi}{2} \right)^3 - 0 \right]} \\ &= \sqrt{\frac{8}{3 \cdot \pi^3} \cdot V_{pk}^2 \cdot \frac{\pi^3}{2^3}} = \sqrt{\frac{1}{3} \cdot V_{pk}^2} = \frac{1}{\sqrt{3}} \cdot V_{pk} \end{aligned}$$

So, $V_{rms} = \frac{1}{\sqrt{3}} \cdot V_{pk} \approx 0.577 V_{pk}$

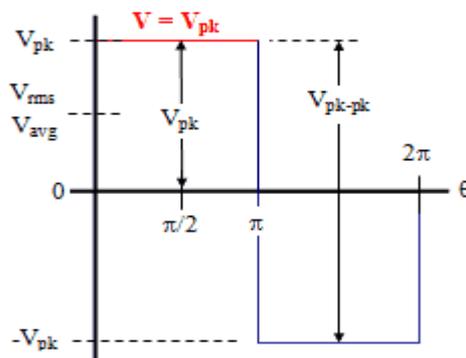
Form factor:

$$\begin{aligned} \text{Form factor of a triangular signal} &= V_{rms} / V_{av} \\ &= .577V_{pk} / .5V_{pk} \\ &= 1.15 \end{aligned}$$

Peak Factor (Or Crest factor): Is defined as the ratio of maximum value to the R.M.S value

$$\begin{aligned} \text{Peak factor of a triangular signal} &= V_{pk} / V_{rms} \\ &= V_{pk} / .577V_{pk} \\ &= 1.732 \end{aligned}$$

Square wave:



When plotted as voltage (V) as a function of phase (θ), a square wave looks similar to the figure to the above. The waveform repeats every 2π radians (360°), and is symmetrical about the voltage axis (when no DC offset is present). Voltage and current exhibiting cyclic behavior is referred to as alternating; i.e., alternating current (AC). One full cycle is shown here.

The basic equation for a square wave is as follows:

$$V_{\text{one complete cycle}} = \begin{cases} 1, & \text{for } 0 \leq \theta < \pi \\ -1, & \text{for } \pi \leq \theta < 2\pi \end{cases}$$

There are a number of ways in which the amplitude of a square wave is referenced, usually as peak voltage (V_{pk} or V_p), peak-to-peak voltage (V_{pp} or V_{p-p} or V_{pkpk} or V_{pk-pk}), average voltage (V_{av} or V_{avg}), and root-mean-square voltage (V_{rms}). Peak voltage and peak-to-peak voltage are apparent by looking at the above plot. Root-mean-square and average voltage are not so apparent.

Average Voltage (V_{avg})

As the name implies, V_{avg} is calculated by taking the average of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the square wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through $\pi/2$ radians (90°).

As with the V_{rms} formula, a full derivation for the V_{avg} formula is given here as well.

$$\begin{aligned} V_{avg} &= \frac{1}{\pi/2} \int_0^{\pi/2} V_{pk} \cdot d\theta = \frac{2}{\pi} \cdot V_{pk} \cdot \theta \Big|_0^{\pi/2} \\ &= \frac{2}{\pi} \cdot V_{pk} \cdot \left(\frac{\pi}{2} - 0 \right) = V_{pk} \end{aligned}$$

$$\text{So, } V_{avg} = V_{pk}$$

Root-Mean-Square Voltage (V_{rms})

As the name implies, V_{rms} is calculated by taking the square root of the mean average of the square of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the square wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through $\pi/2$ radians (90°).

V_{rms} is the value indicated by the vast majority of AC voltmeters. It is the value that, when applied across a resistance, produces that same amount of heat that a direct current (DC) voltage of the same magnitude would produce. For example, 1 V applied across a 1Ω resistor produces 1 W of heat. A 1 V_{rms} square wave applied across a 1Ω resistor also produces 1 W of heat. That 1 V_{rms} square wave has a peak voltage of 1 V, and a peak-to-peak voltage of 2 V.

Since finding a full derivation of the formulas for root-mean-square (V_{rms}) voltage is difficult, it is done here for you.

$$\begin{aligned}
 V_{\text{rms}} &= \sqrt{\frac{1}{\pi/2} \cdot \int_0^{\pi/2} V_{\text{pk}}^2 \cdot d\theta} = \sqrt{\frac{2}{\pi} \cdot V_{\text{pk}}^2 \cdot \theta \Big|_0^{\pi/2}} \\
 &= \sqrt{\frac{2}{\pi} \cdot V_{\text{pk}}^2 \cdot \left(\frac{\pi}{2} - 0\right)} = \sqrt{V_{\text{pk}}^2} = V_{\text{pk}}
 \end{aligned}$$

So, $V_{\text{rms}} = V_{\text{pk}}$

Form factor:

$$\begin{aligned}
 \text{Form factor of a triangular signal} &= V_{\text{rms}} / V_{\text{av}} \\
 &= V_{\text{pk}} / V_{\text{pk}} \\
 &= 1
 \end{aligned}$$

Peak Factor (Or Crest factor):

$$\begin{aligned}
 \text{Peak factor of a triangular signal} &= V_{\text{pk}} / V_{\text{rms}} \\
 &= V_{\text{pk}} / V_{\text{pk}}
 \end{aligned}$$

J notation:

The mathematics used in Electrical Engineering to add together resistances, currents or DC voltages use what are called “real numbers” either as integers or as fractions. But real numbers are not the only kind of numbers we need to use especially when dealing with frequency dependent sinusoidal sources and vectors. As well as using normal or real numbers, Complex Numbers were introduced to allow complex equations to be solved with numbers that are the square roots of negative numbers, $\sqrt{-1}$.

In electrical engineering this type of number is called an “imaginary number” and to distinguish an imaginary number from a real number the letter “j” known commonly in electrical engineering as the j-operator, is used. The letter j is placed in front of a real number to signify its imaginary number operation.

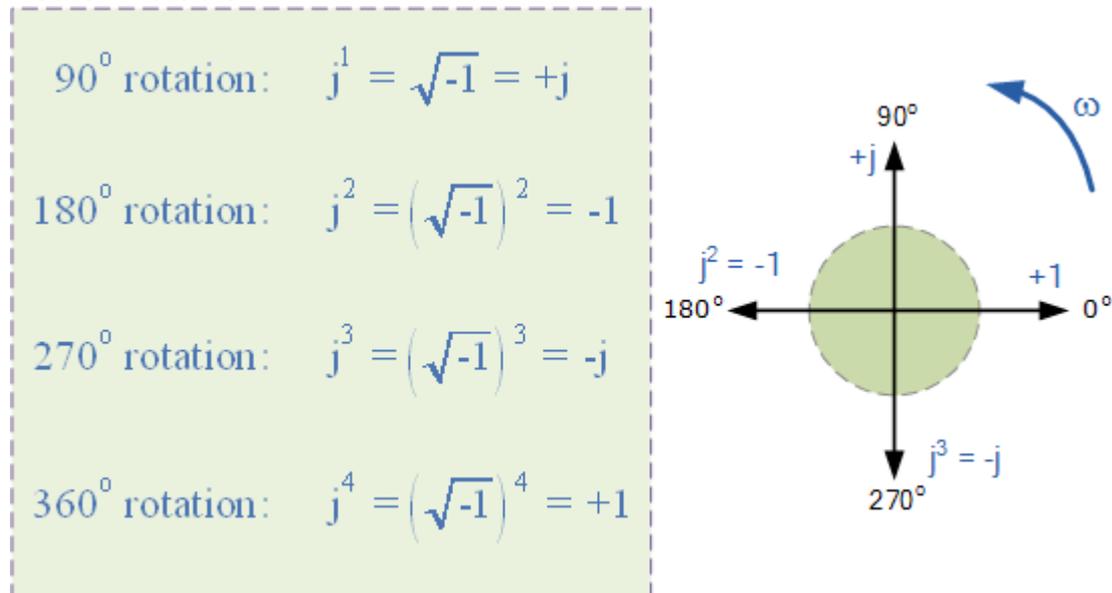
Examples of imaginary numbers are: $j3$, $j12$, $j100$ etc. Then a complex number consists of two distinct but very much related parts, a “Real Number” plus an “Imaginary Number”. Complex Numbers represent points in a two dimensional complex or s-plane that are referenced to two distinct axes. The horizontal axis is called the “real axis” while the vertical axis is called the “imaginary axis”. The real and imaginary parts of a complex number are abbreviated as $\text{Re}(z)$ and $\text{Im}(z)$, respectively.

Complex numbers that are made up of real (the active component) and imaginary (the reactive component) numbers can be added, subtracted and used in exactly the same way as elementary algebra is used to analyse dc circuits. The rules and laws used in mathematics for the addition or subtraction of imaginary numbers are the same as for real numbers, $j2 + j4 = j6$ etc. The only difference is in multiplication because two imaginary numbers multiplied together becomes a negative real number. Real numbers can also be thought of as a complex number but with a zero imaginary part labelled $j0$.

The j-operator has a value exactly equal to $\sqrt{-1}$, so successive multiplication of “j”, ($j \times j$) will result in j having the following values of, -1, -j and +1. As the j-operator is commonly used to indicate the anticlockwise rotation of a vector, each successive multiplication or power of “j”, j^2 , j^3 etc, will force the vector to rotate through an angle of 90° anticlockwise

as shown below. Likewise, if the multiplication of the vector results in a $-j$ operator then the phase shift will be -90° , i.e. a clockwise rotation.

Vector Rotation



So by multiplying an imaginary number by j^2 will rotate the vector by 180° anticlockwise, multiplying by j^3 rotates it 270° and by j^4 rotates it 360° or back to its original position. Multiplication by j^{10} or by j^{30} will cause the vector to rotate anticlockwise by the appropriate amount. In each successive rotation, the magnitude of the vector always remains the same.

Complex and Polar forms of Representation:

In Electrical Engineering there are different ways to represent a complex number either graphically or mathematically. One such way that uses the cosine and sine rule is called the Cartesian or Rectangular Form.

A complex number is represented by a real part and an imaginary part that takes the generalised form of:

$$Z=x+jy$$

Where

Z - is the Complex Number representing the Vector

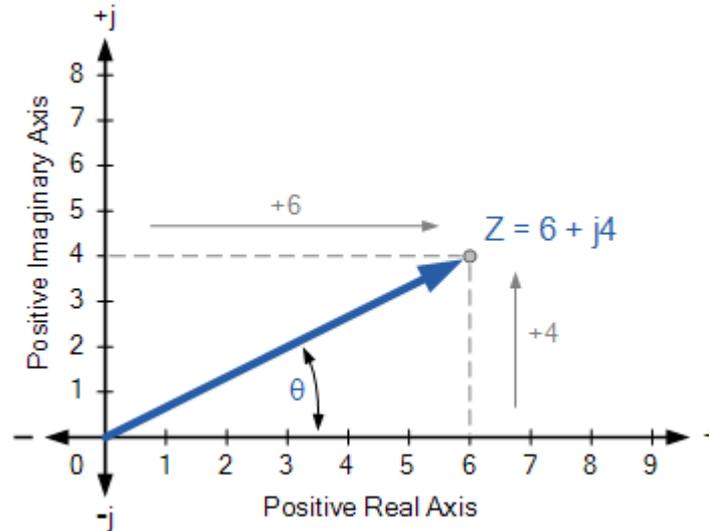
x - is the Real part or the Active component

y - is the Imaginary part or the Reactive component

j - is defined by $\sqrt{-1}$

In the rectangular form, a complex number can be represented as a point on a two-dimensional plane called the complex or s-plane. So for example, $Z = 6 + j4$ represents a single point whose coordinates represent 6 on the horizontal real axis and 4 on the vertical imaginary axis as shown.

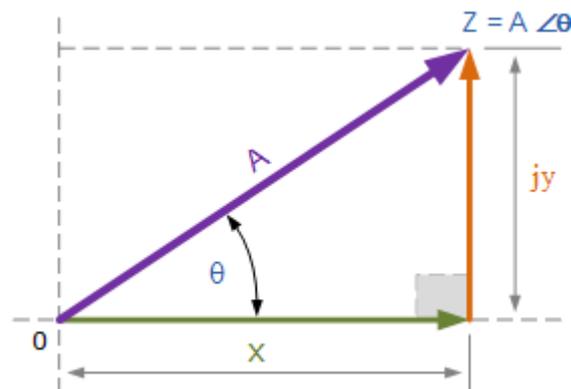
Complex Numbers using the Complex or s-plane:



Complex Numbers using Polar Form:

Unlike rectangular form which plots points in the complex plane, the Polar Form of a complex number is written in terms of its magnitude and angle. Thus, a polar form vector is presented as: $Z = A \angle \theta$, where: Z is the complex number in polar form, A is the magnitude or modulo of the vector and θ is its angle or argument of A which can be either positive or negative. The magnitude and angle of the point still remains the same as for the rectangular form above, this time in polar form the location of the point is represented in a “triangular form” as shown below.

Polar Form Representation of a Complex Number:



As the polar representation of a point is based around the triangular form, we can use simple geometry of the triangle and especially trigonometry and Pythagoras's Theorem on triangles to find both the magnitude and the angle of the complex number. As we remember from school, trigonometry deals with the relationship between the sides and the angles of triangles so we can describe the relationships between the sides as:

$$A^2 = X^2 + Y^2$$

$$A = \sqrt{X^2 + Y^2}$$

$$\text{Also } X = A \cos \theta \quad Y = A \sin \theta$$

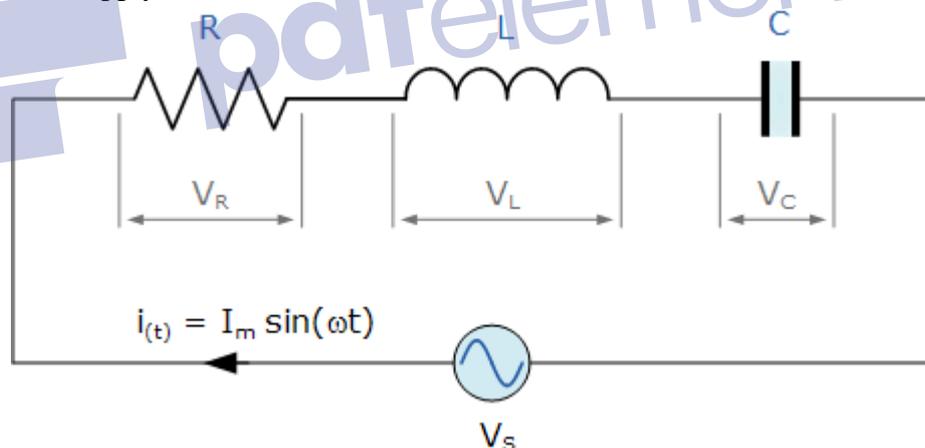
Using trigonometry again, the angle θ of A is given as follows.

$$\theta = \tan^{-1} y/x$$

Then in Polar form the length of A and its angle represents the complex number instead of a point. Also in polar form, the conjugate of the complex number has the same magnitude or modulus it is the sign of the angle that changes, so for example the conjugate of $6 \angle 30^\circ$ would be $6 \angle -30^\circ$.

Steady state Analysis of Series RLC circuits:

Thus far we have seen that the three basic passive components: resistance (R), inductance (L), and capacitance (C) have very different phase relationships to each other when connected to a sinusoidal AC supply.



In a pure ohmic resistor the voltage waveforms are “in-phase” with the current. In a pure inductance the voltage waveform “leads” the current by 90° , giving us the expression of: ELI. In a pure capacitance the voltage waveform “lags” the current by 90° , giving us the expression of: ICE.

This phase difference, Φ depends upon the reactive value of the components being used and hopefully by now we know that reactance, (X) is zero if the circuit element is resistive, positive if the circuit element is inductive and negative if it is capacitive thus giving their resulting impedances as:

Element Impedance:

Circuit element	Resistnsnce(R)	Reactance(X)	Impeadance(Z)
RESISTOR	R	0	$Z_R=R \angle 0^\circ$
INDUCTOR	L	ωL	$Z_L=\omega L \angle 90^\circ$
CAPACITOR	C	$1/\omega C$	$Z_C=1/\omega C \angle -90^\circ$

The series RLC circuit above has a single loop with the instantaneous current flowing through the loop being the same for each circuit element. Since the inductive and capacitive reactance's X_L and X_C are a function of the supply frequency, the sinusoidal response of a series RLC circuit will therefore vary with frequency, f . Then the individual voltage drops across each circuit element of R, L and C element will be "out-of-phase" with each other as defined by:

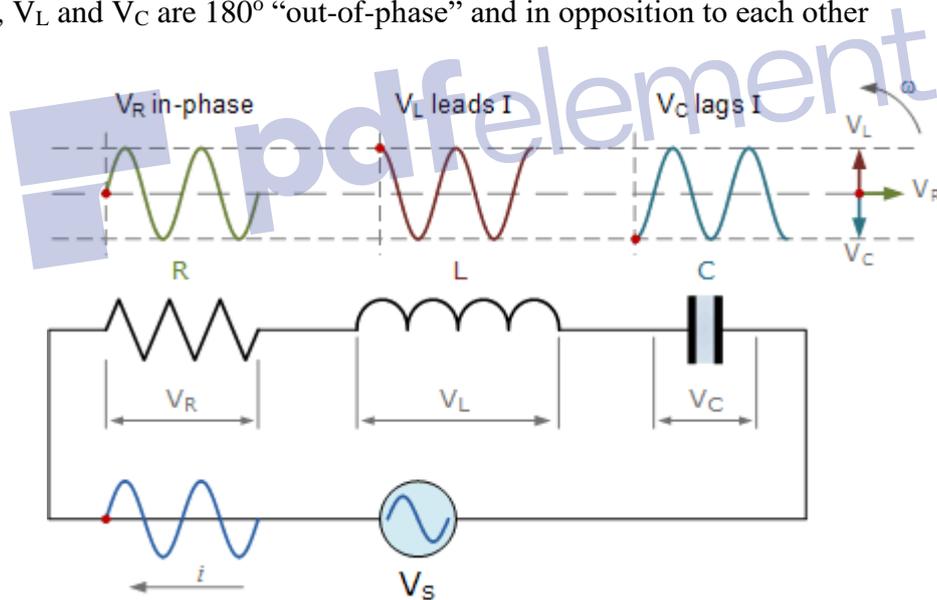
$$i(t) = I_{\max} \sin(\omega t)$$

The instantaneous voltage across a pure resistor, V_R is "in-phase" with current

The instantaneous voltage across a pure inductor, V_L "leads" the current by 90°

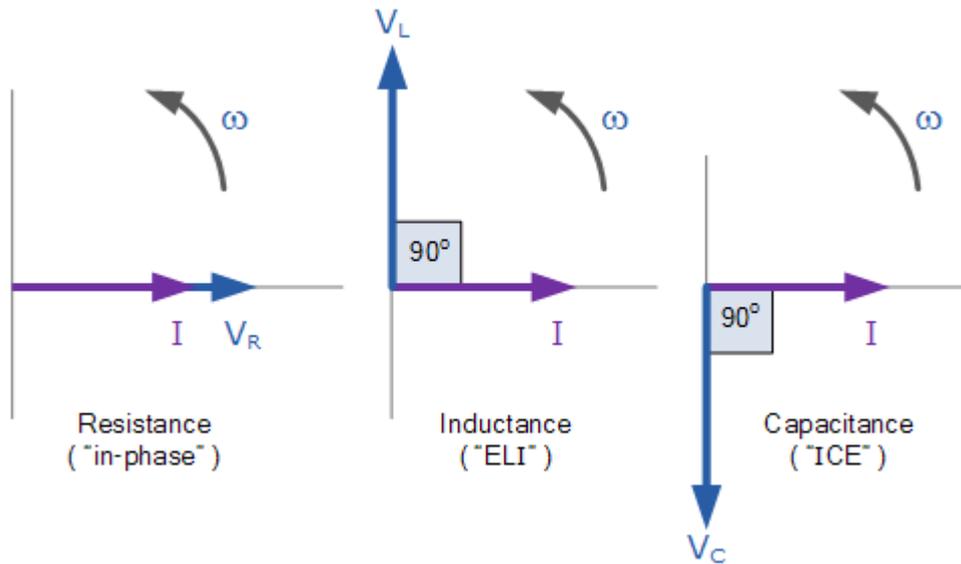
The instantaneous voltage across a pure capacitor, V_C "lags" the current by 90°

Therefore, V_L and V_C are 180° "out-of-phase" and in opposition to each other



The amplitude of the source voltage across all three components in a series RLC circuit is made up of the three individual component voltages, V_R , V_L and V_C with the current common to all three components. The vector diagrams will therefore have the current vector as their reference with the three voltage vectors being plotted with respect to this reference as shown below.

Individual Voltage Vectors



This means then that we cannot simply add together V_R , V_L and V_C to find the supply voltage, V_S across all three components as all three voltage vectors point in different directions with regards to the current vector. Therefore we will have to find the supply voltage, V_S as the Phasor Sum of the three component voltages combined together vectorially.

Kirchoff's voltage law (KVL) for both loop and nodal circuits states that around any closed loop the sum of voltage drops around the loop equals the sum of the EMF's. Then applying this law to the these three voltages will give us the amplitude of the source voltage, V_S as.

Instantaneous Voltages for a Series RLC Circuit:

$$\text{KVL: } V_S - V_R - V_L - V_C = 0$$

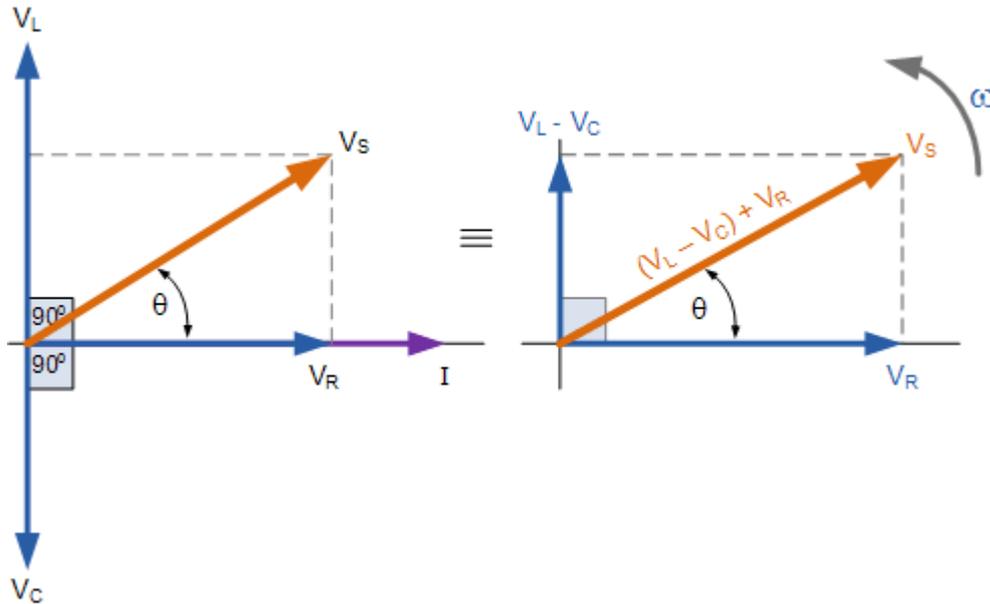
$$V_S - IR - L \frac{di}{dt} - \frac{Q}{C} = 0$$

$$\therefore V_S = IR + L \frac{di}{dt} + \frac{Q}{C}$$

The phasor diagram for a series RLC circuit is produced by combining together the three individual phasors above and adding these voltages vectorially. Since the current flowing through the circuit is common to all three circuit elements we can use this as the reference vector with the three voltage vectors drawn relative to this at their corresponding angles.

The resulting vector V_S is obtained by adding together two of the vectors, V_L and V_C and then adding this sum to the remaining vector V_R . The resulting angle obtained between V_S and i will be the circuits phase angle as shown below.

Phasor Diagram for a Series RLC Circuit:



We can see from the phasor diagram on the right hand side above that the voltage vectors produce a rectangular triangle, comprising of hypotenuse V_S , horizontal axis V_R and vertical axis $V_L - V_C$. Hopefully you will notice then, that this forms our old favourite the Voltage Triangle and we can therefore use Pythagoras's theorem on this voltage triangle to mathematically obtain the value of V_S as shown.

Voltage Triangle for a Series RLC Circuit:

$$V_S^2 = V_R^2 + (V_L - V_C)^2$$

$$V_S = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Please note that when using the above equation, the final reactive voltage must always be positive in value, that is the smallest voltage must always be taken away from the largest voltage we cannot have a negative voltage added to V_R so it is correct to have $V_L - V_C$ or $V_C - V_L$. The smallest value from the largest otherwise the calculation of V_S will be incorrect. We know from above that the current has the same amplitude and phase in all the components of a series RLC circuit. Then the voltage across each component can also be described mathematically according to the current flowing through, and the voltage across each element as.

$$V_R = iR \sin(\omega t + 0^\circ) = i.R$$

$$V_L = iX_L \sin(\omega t + 90^\circ) = i.j\omega L$$

$$V_C = iX_C \sin(\omega t - 90^\circ) = i.\frac{1}{j\omega C}$$

By substituting these values into Pythagoras's equation above for the voltage triangle will give us:

$$V_R = I.R \quad V_L = I.X_L \quad V_C = I.X_C$$

$$V_S = \sqrt{(I.R)^2 + (I.X_L - I.X_C)^2}$$

$$V_S = I.\sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore V_S = I \times Z \quad \text{where: } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

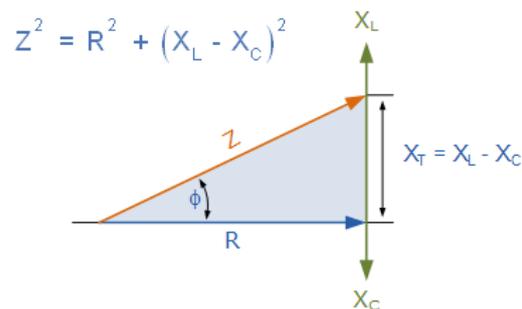
So we can see that the amplitude of the source voltage is proportional to the amplitude of the current flowing through the circuit. This proportionality constant is called the Impedance of the circuit which ultimately depends upon the resistance and the inductive and capacitive reactance's.

Then in the series RLC circuit above, it can be seen that the opposition to current flow is made up of three components, X_L , X_C and R with the reactance, X_T of any series RLC circuit being defined as: $X_T = X_L - X_C$ or $X_T = X_C - X_L$ with the total impedance of the circuit being thought of as the voltage source required to drive a current through it.

The Impedance of a Series RLC Circuit

As the three vector voltages are out-of-phase with each other, X_L , X_C and R must also be "out-of-phase" with each other with the relationship between R , X_L and X_C being the vector sum of these three components thereby giving us the circuits overall impedance, Z . These circuit impedance's can be drawn and represented by an Impedance Triangle as shown below.

The Impedance Triangle for a Series RLC Circuit



The impedance Z of a series RLC circuit depends upon the angular frequency, ω as do X_L and X_C . If the capacitive reactance is greater than the inductive reactance, $X_C > X_L$ then the overall circuit reactance is capacitive giving a leading phase angle.

Likewise, if the inductive reactance is greater than the capacitive reactance, $X_L > X_C$ then the overall circuit reactance is inductive giving the series circuit a lagging phase angle. If the two reactance's are the same and $X_L = X_C$ then the angular frequency at which this occurs is called the resonant frequency and produces the effect of resonance

Then the magnitude of the current depends upon the frequency applied to the series RLC circuit. When impedance, Z is at its maximum, the current is a minimum and likewise, when Z is at its minimum, the current is at maximum. So the above equation for impedance can be re-written as:

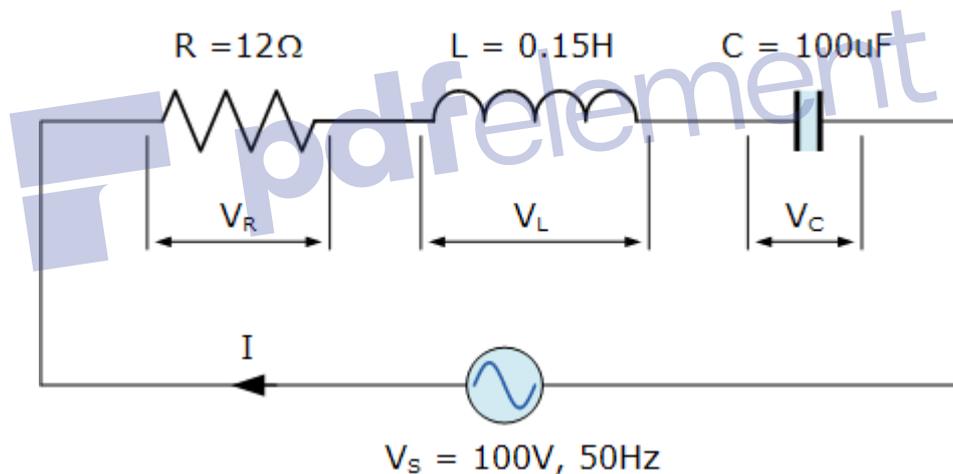
$$\text{Impedance, } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The phase angle, θ between the source voltage, V_s and the current, i is the same as for the angle between Z and R in the impedance triangle. This phase angle may be positive or negative in value depending on whether the source voltage leads or lags the circuit current and can be calculated mathematically from the ohmic values of the impedance triangle as:

$$\cos\phi = \frac{R}{Z} \quad \sin\phi = \frac{X_L - X_C}{Z} \quad \tan\phi = \frac{X_L - X_C}{R}$$

Series RLC Circuit Example

A series RLC circuit containing a resistance of 12Ω , an inductance of 0.15H and a capacitor of $100\mu\text{F}$ are connected in series across a 100V , 50Hz supply. Calculate the total circuit impedance, the circuits current, power factor and draw the voltage phasor diagram



Inductive Reactance, X_L .

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.15 = 47.13\Omega$$

Capacitive Reactance, X_C .

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

Circuit Impedance, Z

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

Circuits Current, I.

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14 \text{ Amps}$$

Voltages across the Series RLC Circuit, V_R , V_L , V_C .

$$V_R = I \times R = 5.14 \times 12 = 61.7 \text{ volts}$$

$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2 \text{ volts}$$

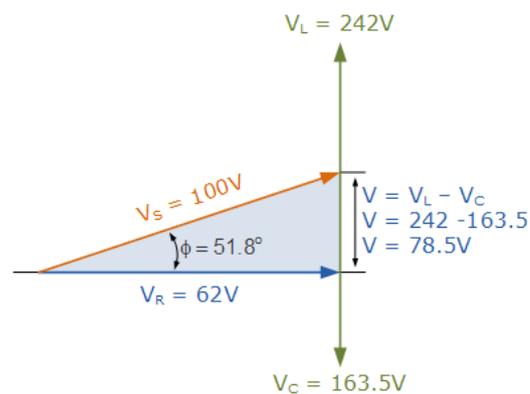
$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5 \text{ volts}$$

Circuits Power factor and Phase Angle, θ .

$$\cos \phi = \frac{R}{Z} = \frac{12}{19.4} = 0.619$$

$$\therefore \cos^{-1} 0.619 = 51.8^\circ \text{ lagging}$$

Phasor Diagram.



Concept of Reactance, Impedance, Susceptance and Admittance:

Reactance is essentially inertia against the motion of electrons. It is present anywhere electric or magnetic fields are developed in proportion to applied voltage or current, respectively; but most notably in capacitors and inductors. When alternating current goes through a pure reactance, a voltage drop is produced that is 90° out of phase with the current. Reactance is mathematically symbolized by the letter “X” and is measured in the unit of ohms (Ω).

Impedance is a comprehensive expression of any and all forms of opposition to electron flow, including both resistance and reactance. It is present in all circuits, and in all components. When alternating current goes through an impedance, a voltage drop is produced that is somewhere between 0° and 90° out of phase with the current. Impedance is mathematically symbolized by the letter “Z” and is measured in the unit of ohms (Ω), in complex form

Admittance is also a complex number as impedance which is having a real part, Conductance (G) and imaginary part, Susceptance (B).

$$Y = G + jB$$

$Y \rightarrow$ Admittance in Siemens

$$G \rightarrow \text{Conductance in Siemens} = \frac{R}{R^2 + X^2}$$

$$B \rightarrow \text{Susceptance in Siemens} = -\frac{X}{R^2 + X^2}$$

(it is negative for capacitive susceptance and positive for inductive susceptance)

$$j^2 = -1$$

$$|Y| = \sqrt{G^2 + B^2} = \frac{1}{\sqrt{R^2 + X^2}}$$

$$\angle Y = \arctan\left(\frac{B}{G}\right) = \arctan\left(-\frac{X}{R}\right)$$

Susceptance (symbolized B) is an expression of the ease with which alternating current (AC) passes through a capacitance or inductance

Phase and phase difference:

Generally all sinusoidal waveforms will not pass exactly through the zero axis point at the same time, but may be “shifted” to the right or to the left of 0° by some value when compared to another sine wave. Any sine wave that does not pass through zero at $t = 0$ has a phase shift.

The phase difference or phase shift as it is also called of a Sinusoidal Waveform is the angle Φ (Greek letter Phi), in degrees or radians that the waveform has shifted from a certain reference point along the horizontal zero axis. In other words phase shift is the lateral

difference between two or more waveforms along a common axis and sinusoidal waveforms of the same frequency can have a phase difference.

The phase difference, Φ of an alternating waveform can vary from between 0 to its maximum time period, T of the waveform during one complete cycle and this can be anywhere along the horizontal axis between, $\Phi = 0$ to 2π (radians) or $\Phi = 0$ to 360° depending upon the angular units used.

Phase difference can also be expressed as a *time shift* of τ in seconds representing a fraction of the time period, T for example, $+10\text{mS}$ or $-50\mu\text{S}$ but generally it is more common to express phase difference as an angular measurement.

Then the equation for the instantaneous value of a sinusoidal voltage or current waveform we developed in the previous Sinusoidal Waveform will need to be modified to take account of the phase angle of the waveform and this new general expression becomes.

Phase Difference Equation

$$A_{(t)} = A_{\text{max}} \times \sin(\omega t \pm \Phi)$$

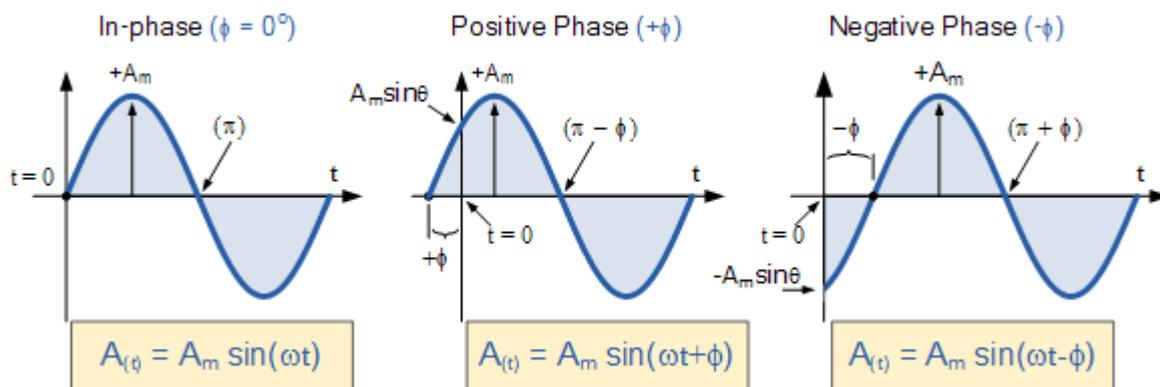
Where:

A_m - is the amplitude of the waveform.

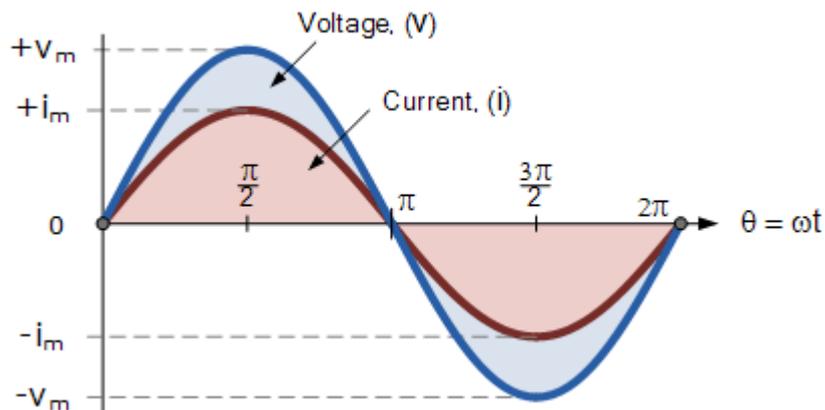
ωt - is the angular frequency of the waveform in radian/sec.

Φ (phi) - is the phase angle in degrees or radians that the waveform has shifted either left or right from the reference point

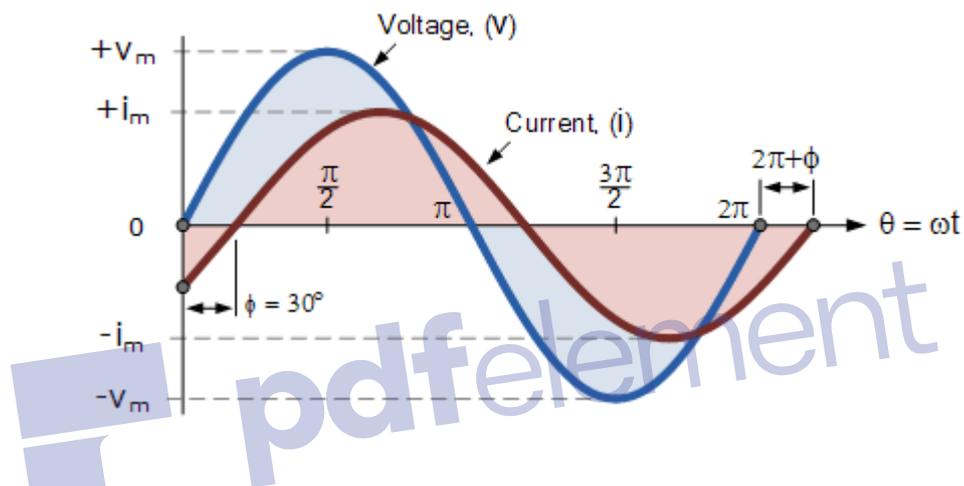
Phase Relationship of a Sinusoidal Waveform:



Two Sinusoidal Waveforms – “in-phase”



Phase Difference of a Sinusoidal Waveform:



The voltage waveform above starts at zero along the horizontal reference axis, but at that same instant of time the current waveform is still negative in value and does not cross this reference axis until 30° later. Then there exists a Phase difference between the two waveforms as the current cross the horizontal reference axis reaching its maximum peak and zero values after the voltage waveform.

As the two waveforms are no longer “in-phase”, they must therefore be “out-of-phase” by an amount determined by phi, Φ and in our example this is 30° . So we can say that the two waveforms are now 30° out-of phase. The current waveform can also be said to be “lagging” behind the voltage waveform by the phase angle, Φ . Then in our example above the two waveforms have a Lagging Phase Difference so the expression for both the voltage and current above will be given as.

$$\text{Voltage, } (v_t) = V_m \sin \omega t$$

$$\text{Current, } (i_t) = I_m \sin(\omega t - \theta)$$

where, i lags v by angle Φ

Likewise, if the current, i has a positive value and crosses the reference axis reaching its maximum peak and zero values at some time before the voltage, v then the current waveform will be “leading” the voltage by some phase angle. Then the two waveforms are said to have a Leading Phase Difference and the expression for both the voltage and the current will be.

$$\text{Voltage, } (v_t) = V_m \sin \omega t$$

$$\text{Current, } (i_t) = I_m \sin(\omega t + \theta)$$

where, i leads v by angle Φ

Concept of power factor, real, reactive and complex power:

Complex Power is defined as the product of Voltage phasor and conjugate of current phasor

If S is the complex power then,

$$S = V \cdot I^*$$

V is the phasor representation of voltage and I^* is the conjugate of current phasor.

So if V is the reference phasor then V can be written as $|V| \angle 0$.

(Usually one phasor is taken reference which makes zero degrees with real axis. It eliminates the necessity of introducing a non zero phase angle for voltage)

Let current lags voltage by an angle ϕ , so $I = |I| \angle -\phi$
(current phasor makes $-\phi$ degrees with real axis)

$$I^* = |I| \angle \phi$$

So,

$$S = |V| |I| \angle (0 + \phi) = |V| |I| \angle \phi$$

(For multiplication of phasors we have considered polar form to facilitate calculation)

Writing the above formula for S in rectangular form we get

$$S = |V| |I| \cos \phi + j |V| |I| \sin \phi$$

The real part of complex power S is $|V| |I| \cos \phi$ which is the real power or average power and the imaginary part $|V| |I| \sin \phi$ is the reactive power.

$$\text{So, } S = P + j Q$$

$$\text{Where } P = |V| |I| \cos \phi \quad \text{and} \quad Q = |V| |I| \sin \phi$$

P is measured in watt and Q is measured in VoltAmp-Reactive or VAR. In power systems instead of these smaller units larger units like Megawatt, MVAR and MVA is used.

The ratio of real power and apparent power is the power factor

$$\begin{aligned}\text{power factor} &= \cos \phi = |P| / |S| \\ &= |P| / \sqrt{P^2 + Q^2}\end{aligned}$$



ELECTRICAL CIRCUITS

UNIT-IV

NETWORK THEOREMS

- **Thevenin's Theorem**
- **Norton's Theorem**
- **Maximum Power Transfer Theorem**
- **Superposition Theorem**
- **Reciprocity Theorem**
- **Tellegen's Theorem**
- **Substitution Theorem**
- **Compensation Theorem**
- **Millman's Theorem**

INTRODUCTION:

Any complicated network i.e. several sources, multiple resistors are present if the single element response is desired then use the network theorems. Network theorems are also can be termed as network reduction techniques. Each and every theorem got its importance of solving network. Let us see some important theorems with DC and AC excitation with detailed procedures.

Thevenin's Theorem and Norton's theorem (Introduction) :

Thevenin's Theorem and Norton's theorem are two important theorems in solving Network problems having many active and passive elements. Using these theorems the networks can be reduced to simple equivalent circuits with one active source and one element. In circuit analysis many a times the current through a branch is required to be found when it's value is changed with all other element values remaining same. In such cases finding out every time the branch current using the conventional mesh and node analysis methods is quite awkward and time consuming. But with the simple equivalent circuits (with one active source and one element) obtained using these two theorems the calculations become very simple. Thevenin's and Norton's theorems are dual theorems.

Thevenin's Theorem Statement :

Any linear, bilateral two terminal network consisting of sources and resistors (Impedance), can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance (Impedance). The equivalent voltage source V_{Th} is the open circuit voltage looking into the terminals (with concerned branch element removed) and the equivalent resistance R_{Th} while all sources are replaced by their internal resistors at ideal condition i.e. voltage source is short circuit and current source is open circuit.

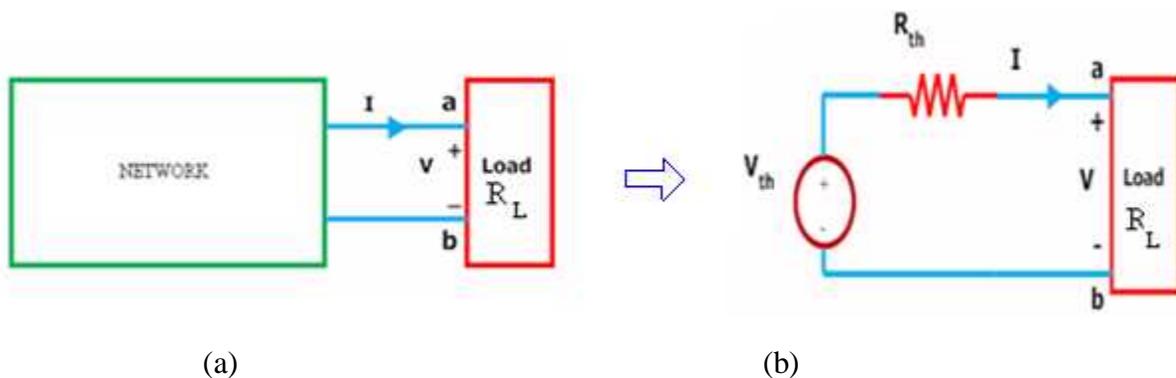
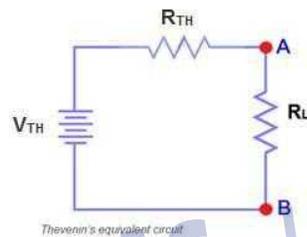


Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance R_L connected across the terminals 'a & b' and figure (b) shows the **Thevenin equivalent circuit** with V_{Th} connected across R_{Th} & R_L .

Main steps to find out V_{Th} and R_{Th} :

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

2. Open circuit voltage V_{OC} across these two terminals is found out using the conventional network mesh/node analysis methods and this would be V_{TH} .
3. **Thevenin resistance R_{TH}** is found out by the method depending upon whether the network contains dependent sources or not.
 - a. With dependent sources: $R_{TH} = V_{oc} / I_{sc}$
 - b. Without dependent sources : $R_{TH} = \text{Equivalent resistance looking into the concerned terminals}$ with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)
4. Replace the network with V_{TH} in series with R_{TH} and the concerned branch resistance (or) load resistance across the load terminals(A&B) as shown in below fig.



Example: Find V_{TH} , R_{TH} and the load current and load voltage flowing through R_L resistor as shown in fig. by using Thevenin's Theorem?

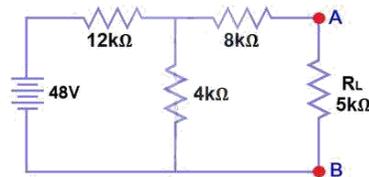


Fig.(a)

Solution:

The resistance R_L is removed and the terminals of the resistance R_L are marked as **A & B** as shown in the fig. (1)

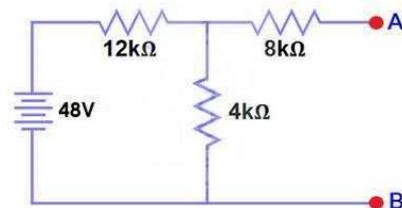
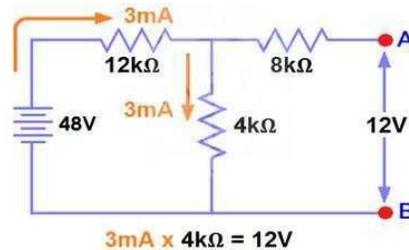


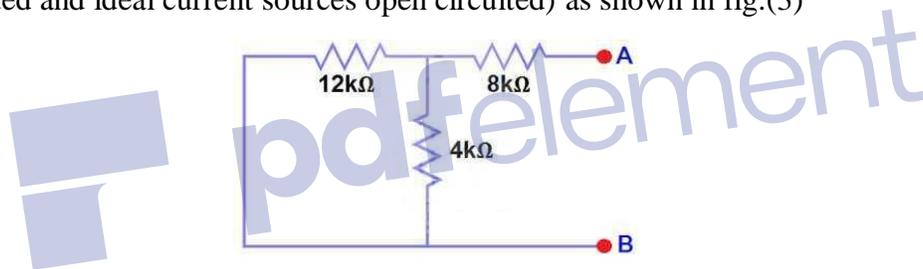
Fig.(1)

Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V_{TH}). We have already removed the load resistor from fig.(a), so the circuit became an open circuit as shown in fig (1). Now we have to calculate the Thevenin's Voltage. Since 3mA Current flows in both 12k Ω and 4k Ω resistors as this is a series circuit because current will not flow in the 8k Ω resistor as it is open. So 12V (3mA x 4k Ω) will appear across the 4k Ω resistor. We also know that current is not flowing through the 8k Ω resistor as it is open circuit, but the 8k Ω resistor is in parallel with 4k resistor. So the same voltage (i.e. 12V) will appear across the 8k Ω resistor as 4k Ω resistor. Therefore 12V will appear across the AB terminals. So, $V_{TH} = 12V$



Fig(2)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) as shown in fig.(3)



Fig(3)

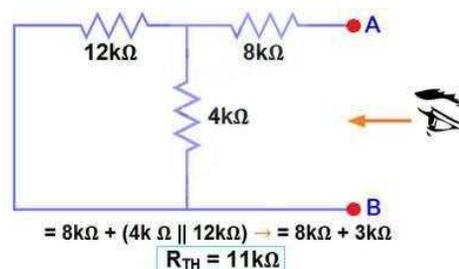
Calculate /measure the Open Circuit Resistance. This is the Thevenin Resistance (R_{TH}) We have Reduced the 48V DC source to zero is equivalent to replace it with a short circuit as shown in figure (3) We can see that 8k Ω resistor is in series with a parallel connection of 4k Ω resistor and 12k Ω resistor. i.e.:

$$8k\Omega + (4k\Omega \parallel 12k\Omega) \dots (\parallel = \text{in parallel with})$$

$$R_{TH} = 8k\Omega + [(4k\Omega \times 12k\Omega) / (4k\Omega + 12k\Omega)]$$

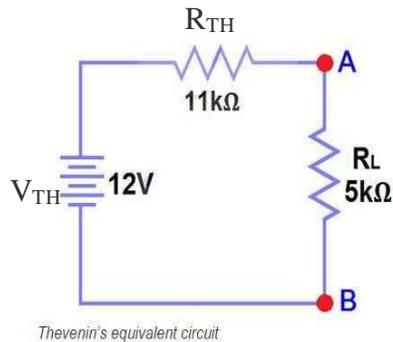
$$R_{TH} = 8k\Omega + 3k\Omega$$

$$R_{TH} = 11k\Omega$$



Fig(4)

Connect the R_{TH} in series with Voltage Source V_{TH} and re-connect the load resistor across the load terminals(A&B) as shown in fig (5) i.e. Thevenin circuit with load resistor. This is the Thevenin's equivalent circuit



Thevenin's equivalent circuit.

Fig(5)

Now apply Ohm's law and calculate the total load current from fig 5.

$$I_L = V_{TH} / (R_{TH} + R_L) = 12V / (11k\Omega + 5k\Omega) = 12/16k\Omega$$

$$I_L = 0.75mA$$

$$\text{And } V_L = I_L \times R_L = 0.75mA \times 5k\Omega$$

$$V_L = 3.75V$$

Norton's Theorem Statement :

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance), can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (Impedance), the current source being the short circuited current across the load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals.

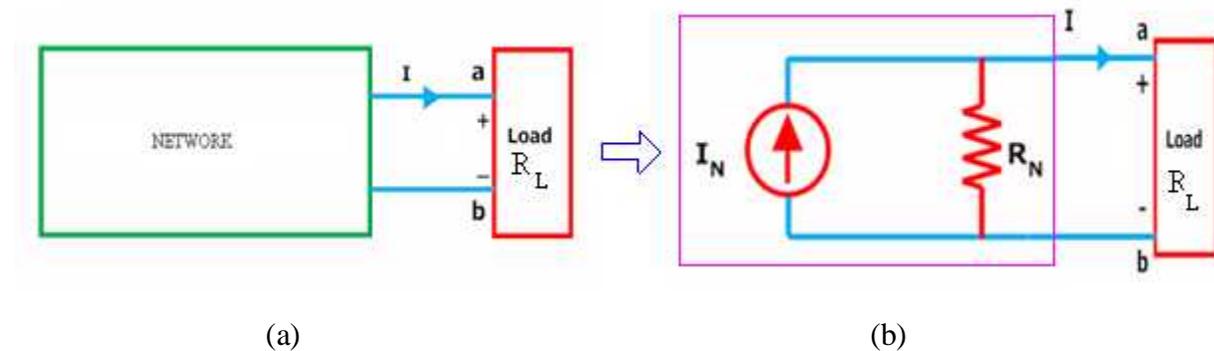
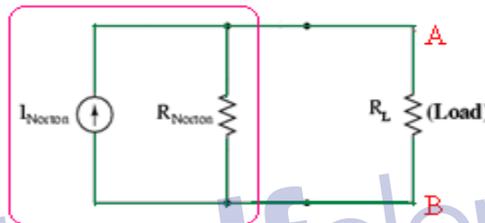


Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance R_L connected across the terminals 'a & b' and figure (b) shows the **Norton equivalent circuit** with I_N connected across R_N & R_L .

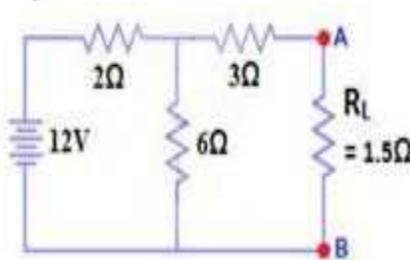
Main steps to find out I_N and R_N :

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

2. Open circuit voltage V_{OC} across these two terminals and I_{SC} through these two terminals are found out using the conventional network mesh/node analysis methods and they are same as what we obtained in Thevenin's equivalent circuit.
3. Next **Norton resistance** R_N is found out depending upon whether the network contains dependent sources or not.
 - a) With dependent sources: $R_N = V_{oc} / I_{sc}$
 - b) Without dependent sources : $R_N =$ Equivalent resistance looking into the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)
4. Replace the network with I_N in parallel with R_N and the concerned branch resistance across the load terminals(A&B) as shown in below fig

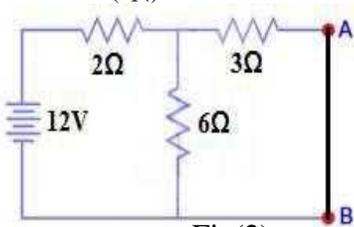


Example: Find the current through the resistance R_L (1.5Ω) of the circuit shown in the figure (a) below using Norton's equivalent circuit.?



Fig(a)

Solution: To find out the Norton's equivalent ckt we have to find out $I_N = I_{sc}$, $R_N = V_{oc} / I_{sc}$. Short the 1.5Ω load resistor as shown in (Fig 2), and Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).



Fig(2)

We have shorted the AB terminals to determine the Norton current, I_N . The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω . So the Total Resistance of the circuit to the Source is:-

$$2\Omega + (6\Omega \parallel 3\Omega) \dots \dots (\parallel = \text{in parallel with})$$

$$R_T = 2\Omega + [(3\Omega \times 6\Omega) / (3\Omega + 6\Omega)]$$

$$R_T = 2\Omega + 2\Omega$$

$$R_T = 4\Omega$$

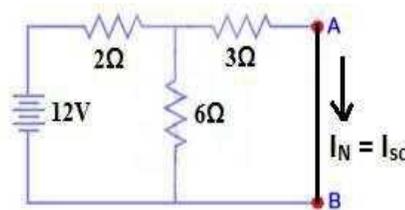
$$I_T = V / R_T$$

$$I_T = 12V / 4\Omega = 3A..$$

Now we have to find $I_{SC} = I_N$... Apply CDR... (Current Divider Rule)...

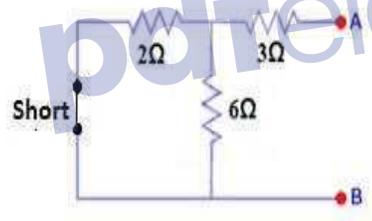
$$I_{SC} = I_N = 3A \times [(6\Omega / (3\Omega + 6\Omega))] = 2A.$$

$$I_{SC} = I_N = 2A.$$



Fig(3)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) and Open Load Resistor. as shown in fig.(4)



Fig(4)

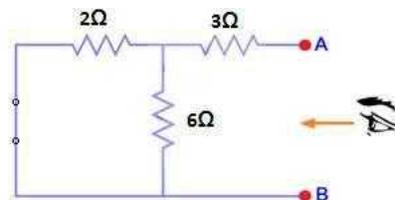
Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N) We have Reduced the 12V DC source to zero is equivalent to replace it with a short circuit as shown in fig(4), We can see that 3Ω resistor is in series with a parallel combination of 6Ω resistor and 2Ω resistor. i.e.:

$$3\Omega + (6\Omega \parallel 2\Omega) \dots \dots (\parallel = \text{in parallel with})$$

$$R_N = 3\Omega + [(6\Omega \times 2\Omega) / (6\Omega + 2\Omega)]$$

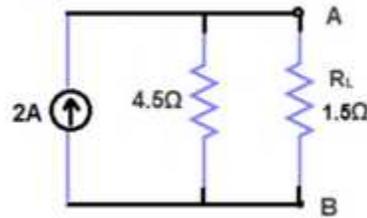
$$R_N = 3\Omega + 1.5\Omega$$

$$R_N = 4.5\Omega$$



Fig(5)

Connect the R_N in Parallel with Current Source I_N and re-connect the load resistor. This is shown in fig (6) i.e. Norton Equivalent circuit with load resistor.



Fig(6)

Now apply the Ohm's Law and calculate the load current through Load resistance across the terminals A&B. Load Current through Load Resistor is

$$I_L = I_N \times [R_N / (R_N + R_L)]$$

$$I_L = 2A \times (4.5\Omega / 4.5\Omega + 1.5k\Omega)$$

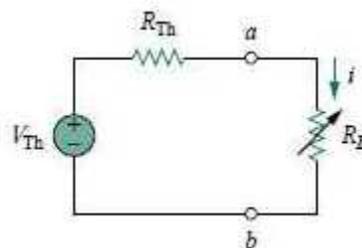
$$I_L = 1.5A \quad I_L = 1.5A$$

Maximum Power Transfer Theorem:

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for Efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. electrical applications with electrical loads such as Loud speakers, antennas, motors etc. it would be required to find out the condition under which maximum power would be transferred from the circuit to the load.

Maximum Power Transfer Theorem Statement:

Any linear, bilateral two terminal network consisting of a resistance load, being connected to a dc network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.



According to Maximum Power Transfer Theorem, for maximum power transfer from the network to the load resistance, R_L must be equal to the source resistance i.e. Network's Thevenin equivalent resistance R_{Th} . i.e. $R_L = R_{Th}$

The load current I in the circuit shown above is given by,

$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

The power delivered by the circuit to the load:

$$P = I^2 R = \frac{V_{TH}^2}{(R_{TH} + R_L)^2} R_L$$

The condition for maximum power transfer can be obtained by differentiating the above expression for power delivered with respect to the load resistance (Since we want to find out the value of R_L for maximum power transfer) and equating it to zero as :

$$\frac{\partial P}{\partial R_L} = 0 = \frac{V_{TH}^2}{(R_{TH} + R_L)^2} - \frac{2V_{TH}^2}{(R_{TH} + R_L)^3} R_L = 0$$

Simplifying the above equation, we get:

$$(R_{TH} + R_L) - 2R_L = 0 \Rightarrow R_L = R_{TH}$$

Under the condition of maximum power transfer, the power delivered to the load is given by :

$$P_{MAX} = \frac{V_{TH}^2}{(R_L + R_L)^2} \times R_L = \frac{V_{TH}^2}{4R_L}$$

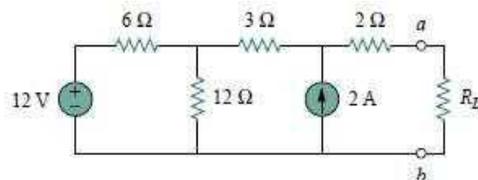
Under the condition of maximum power transfer, the efficiency η of the network is then given by:

$$P_{LOSS} = \frac{V_{TH}^2}{(R_L + R_L)^2} \times R_{TH} = \frac{V_{TH}^2}{4R_L}$$

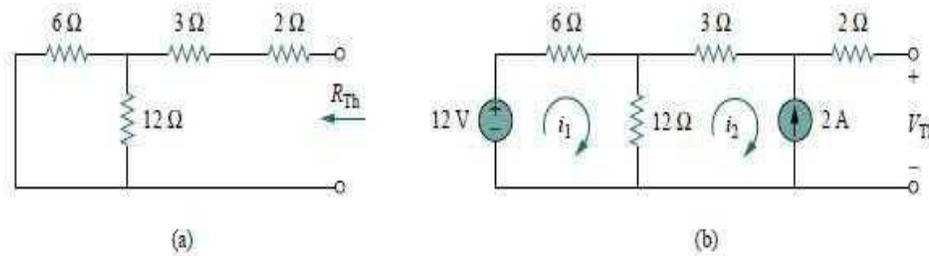
$$\eta = \frac{\text{output}}{\text{input}} = \frac{\frac{V_{TH}^2}{4R_L}}{\left(\frac{V_{TH}^2}{4R_L} + \frac{V_{TH}^2}{4R_L}\right)} = 0.50$$

For maximum power transfer the load resistance should be equal to the Thevenin equivalent resistance (or Norton equivalent resistance) of the network to which it is connected . Under the condition of maximum power transfer the efficiency of the system is 50 %.

Example: Find the value of R_L for maximum power transfer in the circuit of Fig. Find the maximum power.?



Solution: We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals $a-b$. To get R_{Th} , we use the circuit in Fig. (a)



$$R_{Th} = 2 + 3 + (6 // 12) = 5 + \left(\frac{6 \times 12}{6 + 12}\right) = 5 + 4 = 9\ \Omega$$

To get V_{Th} , we consider the circuit in Fig.(b). Applying mesh analysis,

$$-12 + 18i_1 - 12i_2 = 0,$$

$$i_2 = -2\text{ A},$$

Solving for i_1 , we get $i_1 = -2/3$.

Applying KVL around the outer loop to get V_{Th} across terminals $a-b$, we obtain,

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$$

$$V_{Th} = 22\text{ V}$$

For maximum power transfer, $R_L = R_{Th} = 9\ \Omega$ and the maximum power is,

$$P_{MAX} = \frac{V_{TH}^2}{4R_L} = \frac{22 \times 22}{4 \times 9} = 13.44\text{ W}$$

Superposition Theorem:

The principle of superposition helps us to analyze a linear circuit with more than one current or voltage sources sometimes it is easier to find out the voltage across or current in a branch of the circuit by considering the effect of one source at a time by replacing the other sources with their ideal internal resistances.

Superposition Theorem Statement:

Any linear, bilateral two terminal network consisting of more than one sources, The total current or voltage in any part of a network is equal to the algebraic sum of the currents or voltages in the required branch with each source acting individually while other sources are replaced by their ideal internal resistances. (i.e. Voltage sources by a short circuit and current sources by open circuit)

Steps to Apply Super position Principle:

1. Replace all independent sources with their internal resistances except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Example: By Using the superposition theorem find I in the circuit shown in figure?

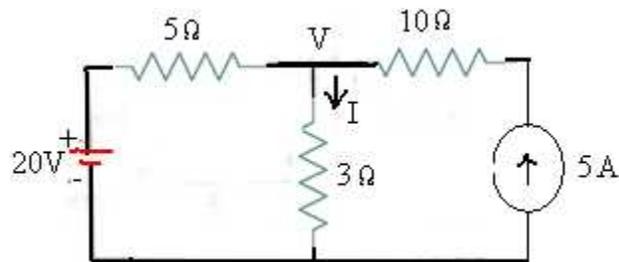


Fig.(a)

Solution: Applying the superposition theorem, the current I_2 in the resistance of $3\ \Omega$ due to the voltage source of 20V alone, with current source of 5A open circuited [as shown in the figure.1 below] is given by :

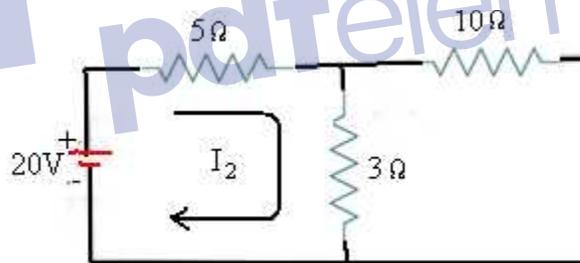


Fig1

$$I_2 = 20/(5+3) = 2.5\text{A}$$

Similarly the current I_5 in the resistance of $3\ \Omega$ due to the current source of 5A alone with voltage source of 20V short circuited [as shown in the figure.2 below] is given by :

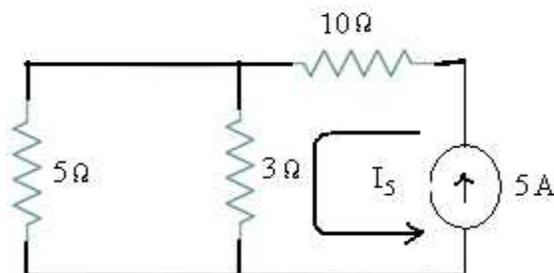


Fig.2

$$I_5 = 5 \times 5 / (3+5) = 3.125 \text{ A}$$

The total current passing through the resistance of 3Ω is then $= I_2 + I_5 = 2.5 + 3.125 = 5.625 \text{ A}$

Let us verify the solution using the basic nodal analysis referring to the node marked with V in fig.(a). Then we get :

$$\frac{V - 20}{5} + \frac{V}{3} = 5$$

$$3V - 60 + 5V = 15 \times 5$$

$$8V - 60 = 75$$

$$8V = 135$$

$$V = 16.875$$

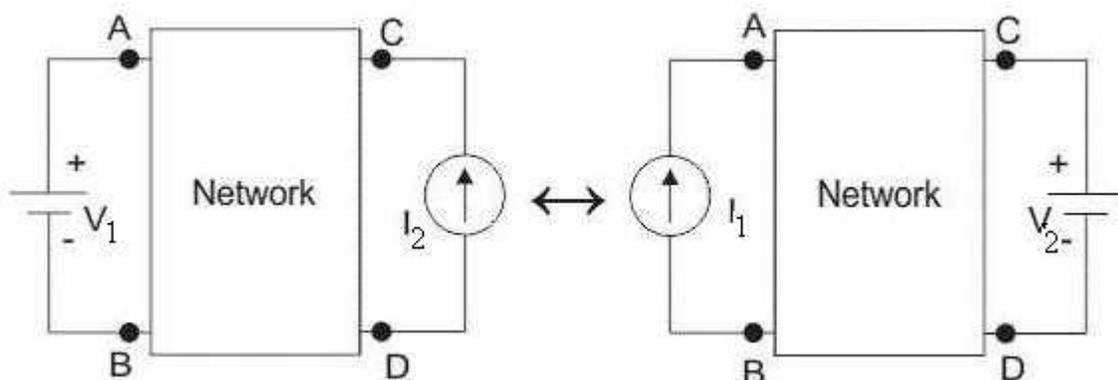
The current I passing through the resistance of $3\Omega = V/3 = 16.875/3 = 5.625 \text{ A}$.

Reciprocity theorem:

Under Basic Electrical Engineering In many electrical networks it is found that if the positions of voltage source and ammeter are interchanged, the reading of ammeter remains the same. Suppose a voltage source is connected to a passive network and an ammeter is connected to other part of the network to indicate the response. Now any one interchanges the positions of ammeter and voltage source that means he or she connects the voltage source at the part of the network where the ammeter was connected and connects ammeter to that part of the network where the voltage source was connected. The response of the ammeter means current through the ammeter would be the same in both the cases. This is where the property of reciprocity comes in the circuit. The particular circuit that has this reciprocal property, is called reciprocal circuit.

Reciprocity theorem Statement:

Any linear, bilateral two terminal network the ratio of excitation to response is constant even though the source is interchanged from input terminals to the output terminals.



$$\frac{V_1}{I_2} = \frac{V_2}{I_1}$$

Steps For Solution Of a Network Utilizing Reciprocity Theorem:

1. The branches between which reciprocity is to be established to be selected first.
2. The current in the branch is obtained using conventional network analysis.
3. The voltage source is interchanged between the branches concerned.
4. The current in the branch where the voltage source was existing earlier is calculated.

It may observe that the currents obtained in 2 & 4 are identical to each other.

Example: Verify the reciprocity theorem for the network shown in the figure (1).?

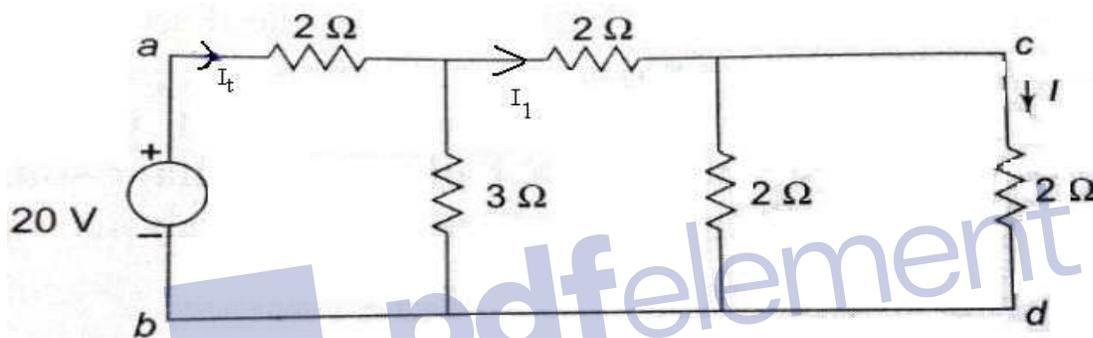


Fig.(1)

Solution: Total resistance in the circuit across the applied voltage of 20 V is

$$\begin{aligned} R_{TH} &= 2 + [3 \parallel (2 + (2 \parallel 2))] \\ &= 2 + [3 \parallel 3] \\ &= 3.5 \Omega \end{aligned}$$

The total current drawn by the circuit $I_T = \frac{V}{R_{TH}} = 20/3.5 = 5.71 \text{ A}$

The current **I** in the branch 'cd' with 2 Ω resistance is find by using current division rule. For that first find I_1 current.

$$I_1 = 5.71 \times \frac{3}{3+3} = 2.855 \text{ A}$$

The current **I** in the 'cd' branch is

$$I = 2.855 \times \frac{2}{2+2} = 1.427 \text{ A}$$

Now the source voltage and the response are interchanged between branches 'ab' and 'cd' as shown in the figure (2) below

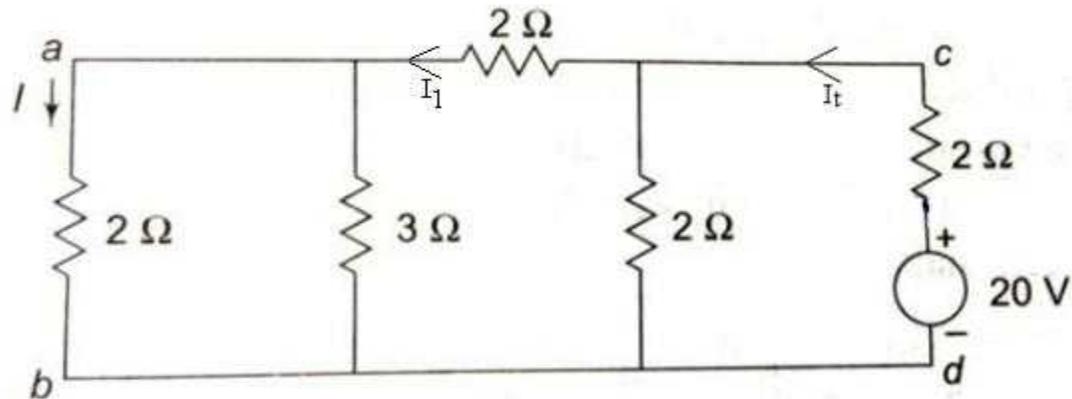


Fig.(2)

Total resistance in the circuit across the applied voltage of 20 V is

$$R_{TH} = 2 + [2 \parallel (2 + (2 \parallel 3))]$$

$$= 2 + [2 \parallel 3.2]$$

$$= 3.23 \Omega$$

The total current drawn by the circuit $I_T = \frac{V}{R_{TH}} = 20/3.23 = 6.19 \text{ A}$

The current I in the branch 'ab' with 2 Ω resistance is found by using current division rule. For that first find I_1 current.

$$I_1 = 6.19 \times \frac{2}{3.2 + 2} = 2.38 \text{ A}$$

The current I in the 'ab' branch is

$$I = 2.38 \times \frac{3}{3 + 2} = 1.427 \text{ A}$$

The current in the branch 'ab' = **1.427 A** which is same as the current we got in branch 'cd' when the voltage was given from branch 'ab'. Thus the **reciprocity theorem** is verified.

Tellegen's Theorem:

This theorem is the one of the most general theorems in network analysis regardless to the type and nature, Tellegen's theorem is applicable to any network made up of lumped two terminal elements.

Tellegen's Theorem Statement:

In any linear, non-linear, passive, active, time variant or time invariant network the algebraic sum of power at any given instant is zero. Thus for K^{th} branch, this theorem states that

$$\sum_{K=1}^n v_K i_K = 0$$

Where, n =Being the number of branches,

V_K =Voltage drop in the branch,

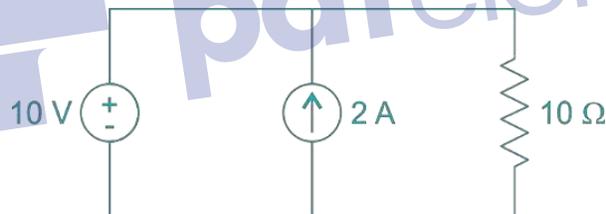
I_K =Current drop in the branch.

It also evident that the sum of power delivered to the network is equal to the sum of power absorbed by the network elements.

Steps For Solution Of a Network Utilizing Tellegen's Theorem:

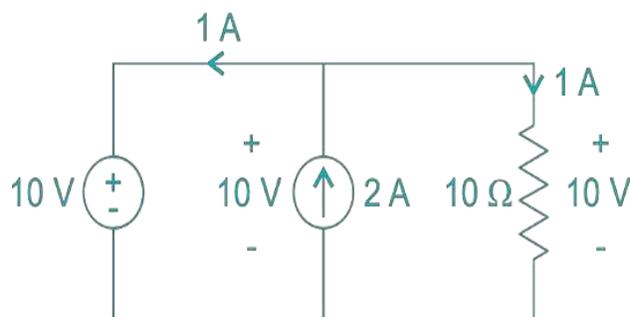
1. Find the branch voltage drops and corresponding branch currents using conventional analysis.
2. Summing all products of branch voltage and current.

Example: Verify the Tellegen's theorem for the given circuit.?



Solution: If current flows from + to – then treat it as power absorption.

If current flow from – to + then treat it as power delivering.



$$\therefore P_{10V} = V \cdot I = 10 \times 1 = 10 \text{ watt (P}_{\text{absorbed}}\text{)}.$$

$$P_{2A} = V \cdot I = 10 \times 2 = 20 \text{ watt (} P_{\text{delivered}} \text{)}$$

$$P_{10\Omega} = I^2 \cdot R = 1^2 \times 10 = 10 \text{ watt (} P_{\text{absorbed}} \text{)}$$

$$P_{\text{delivered}} = P_{\text{absorbed}} = 20 \text{ watt}$$

Hence Tellegen's theorem is verified.

Substitution Theorem Statement:

The voltage across and the current through a Branch in a bilateral network is known, the branch can be replaced by any combination of elements in such a way that the same voltage will appear across and same current will pass through the chosen terminals. In other words for branch equivalence the terminal voltage and the current must be same.

This is illustrated with a simple circuit shown in the figure below.

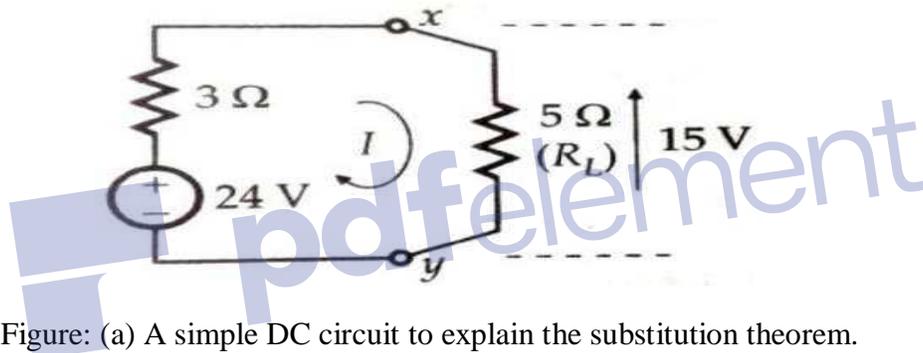


Figure: (a) A simple DC circuit to explain the substitution theorem.

In this circuit the load resistance R_L is the branch being considered for equivalence. The current I through the load resistance $R_L = 24/(3+5) = 3 \text{ A}$.

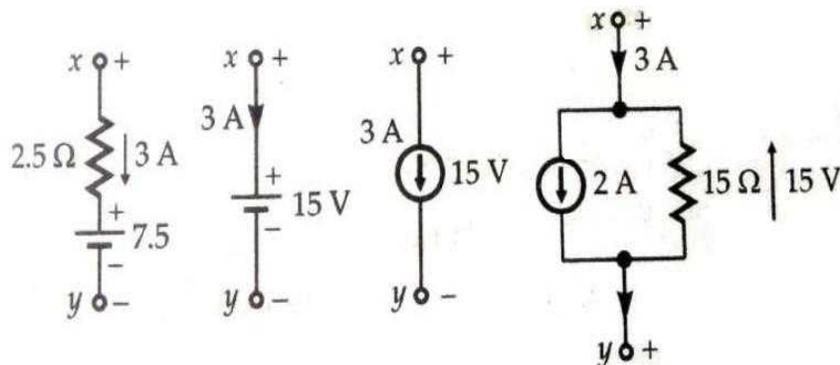


Figure: (b) Equivalent branches across terminals 'xy'

In figure (b) above several equivalents of branch 'xy' are shown. It may be noted that in all the cases the terminal voltage across and the current through the equivalent branch are the same as that of the original branch R_L . It may also be observed that a known potential

difference and a current in a branch can be replaced by an ideal voltage source or an ideal current source respectively.

The limitation of this theorem is that it cannot be used to solve a network containing two or more sources that are not in series or parallel.

Example: Using substitution theorem, draw equivalent branches for the branch 'a-b' of the network of Fig.(a)?

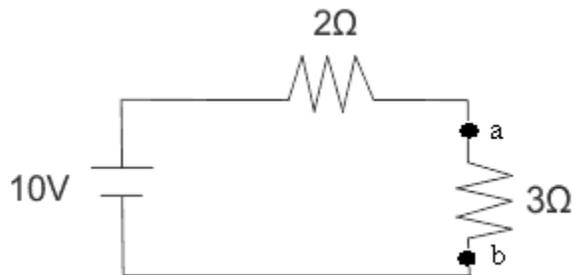


Fig.(a)

Solution: As per voltage division rule voltage across 3Ω and 2Ω resistance are

$$V_{3\Omega} = \frac{10 \times 3}{2 + 3} = 6V$$

$$V_{2\Omega} = \frac{10 \times 2}{2 + 3} = 4V$$

$$\text{Current through the circuit is, } I = \frac{10}{2+3} = 2A$$

If we replace the 3Ω resistance with a voltage source of 6 V as shown in fig (1), then

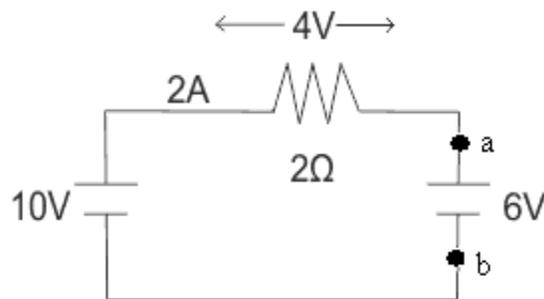


Fig.(1)

Then according to Ohm's Law the voltage across 2Ω resistance and current through the circuit is,

$$V_{2\Omega} = 10 - 6 = 4V$$

$$I = \frac{10 - 6}{2} = 2A$$

Alternately if we replace 3Ω resistance with a current source of $2A$ as shown in Fig(2), then

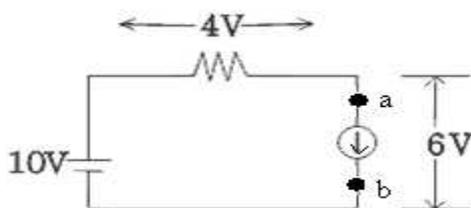


Fig.(2)

Voltage across 2Ω is $V_{2\Omega} = 10 - (3 \times 2) = 4 \text{ V}$ and

Voltage across $2A$ current source is $V_{2A} = 10 - 4 = 6 \text{ V}$.

The voltage across 2Ω resistance and current through the circuit is unaltered i.e. all initial condition of the circuit is intact.

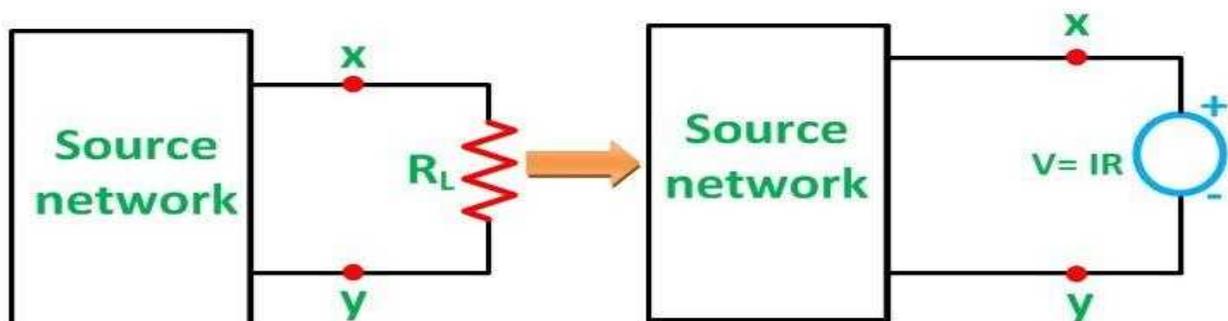
Compensation Theorem:

It is one of the important theorems in Network Analysis, which finds its application mostly in calculating the sensitivity of electrical networks & bridges and solving electrical networks. In many circuits, after the circuit is analyzed, it is realized that only a small change need to be made to a component to get a desired result. In such a case we would normally have to recalculate. The compensation theorem allows us to compensate properly for such changes without sacrificing accuracy.

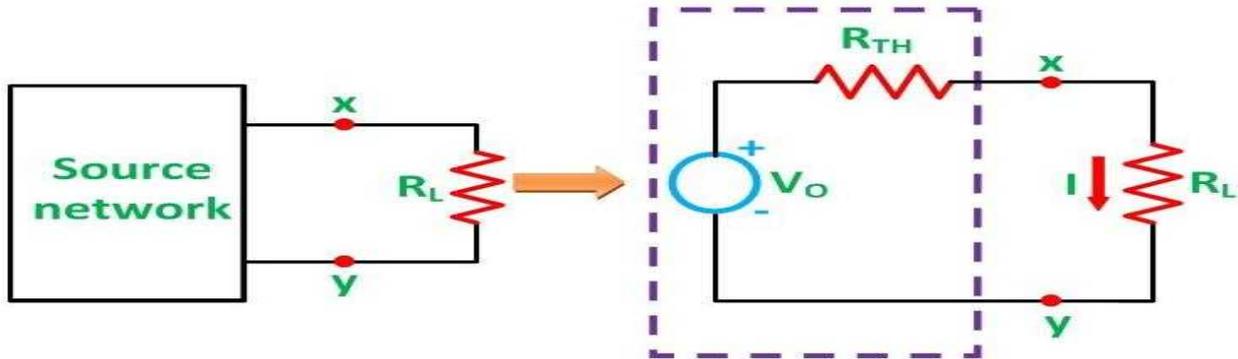
Compensation Theorem Statement:

In a linear, bilateral, time invariant network when the resistance (R) of an uncoupled branch, carrying a current (I), is changed by (ΔR). The currents in all the branches would change and can be obtained by assuming that an ideal voltage source of (V_C) has been connected such that $V_C = I (\Delta R)$ in series with ($R + \Delta R$) when all other sources in the network are replaced by their internal resistances.

In Compensation Theorem, the source voltage (V_C) opposes the original current. In simple words compensation theorem can be stated as – the resistance of any network can be replaced by a voltage source, having the same voltage as the voltage drop across the resistance which is replaced.



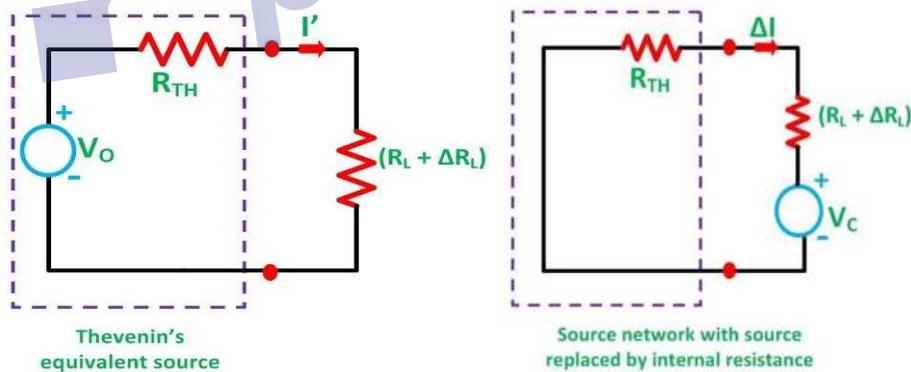
Let us assume a load R_L be connected to a DC source network whose Thevenin's equivalent gives V_0 as the Thevenin's voltage and R_{TH} as the Thevenin's resistance as shown in the figure below.



Here,

$$I = \frac{V_0}{R_{TH} + R_L} \dots \dots \dots (1)$$

Let the load resistance R_L be changed to $(R_L + \Delta R_L)$. Since the rest of the circuit remains unchanged, the Thevenin's equivalent network remains the same as shown in the circuit diagram below



Here,

$$I' = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} \dots \dots \dots (2)$$

The change of current being termed as ΔI Therefore,

$$\Delta I = I' - I \dots \dots \dots (3)$$

Putting the value of I' and I from the equation (1) and (2) in the equation (3) we will get the following equation.

$$\Delta I = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} - \frac{V_0}{R_{TH} + R_L}$$

$$\Delta I = \frac{V_0 \{ (R_{TH} + R_L) - (R_{TH} + (R_L + \Delta R_L)) \}}{(R_{TH} + (R_L + \Delta R_L)) \times (R_{TH} + R_L)}$$

$$\Delta I = - \left[\frac{V_0}{R_{TH} + R_L} \right] \frac{R_{TH}}{R_{TH} + (R_L + \Delta R_L)} \dots \dots \dots (4)$$

Now, putting the value of I from the equation (1) in the equation (4), we will get the following equation.

$$I = - \frac{I R_{TH}}{R_{TH} + (R_L + \Delta R_L)} \dots \dots \dots (5)$$

As we know, $V_C = I \Delta R_L$ and is known as compensating voltage. Therefore, the equation (5) becomes.

$$\Delta I = \frac{-V_C}{R_{TH} + (R_L + \Delta R_L)}$$

Hence, Compensation Theorem tells that with the change of branch resistance, branch currents changes and the change is equivalent to an ideal compensating voltage source in series with the branch opposing the original current, all other sources in the network being replaced by their internal resistances.

Example: Determine the current flowing through the ammeter having an internal resistance of 1Ω connected in series with a 3Ω resistor as shown in the fig (a).?

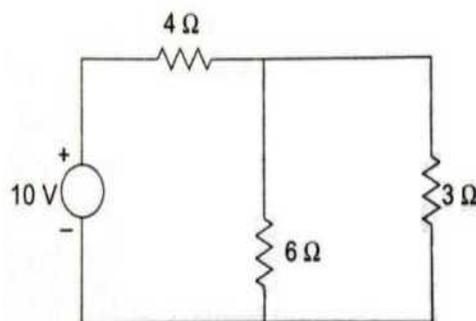
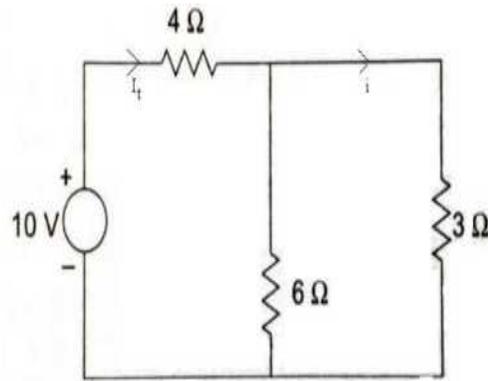


Fig.(a)

Solution:



The current flowing through the 3 Ω branch i,

$$i = I_t \left[\frac{6}{6+3} \right]$$

$$I_t = \frac{10}{4 + (6 \parallel 3)}$$

$$I_t = \frac{10}{4+2}$$

$$I_t = 1.67 \text{ A}$$

$$i = 1.67 \left[\frac{6}{6+3} \right]$$

$$i = 1.11 \text{ A}$$

Now when we connect the ammeter with an internal resistance of 1 Ω in the 3 Ω branch, there is a change in resistance. This change in resistance causes currents in other branches as if a voltage source of voltage v is

$$V = i \cdot R = 1.11 \times 1 = 1.11 \text{ v}$$

1.11 v is inserted in the 3 Ω branch as shown in the fig (1) below.

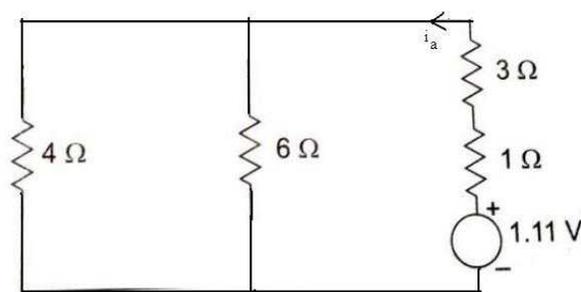


Fig.(1)

Current due to this additional source of 1.11 V in the 3 Ω branch i_a is,

$$i_a = \frac{1.11}{(1+3+(6\parallel 4))}$$

$$i_a = \frac{1.11}{(1+3+2.4)}$$

$$i_a = 0.17A$$

This current flows in the opposite direction to that of the original current i through the 3 Ω branch (i.e. i_a is opposite to i)

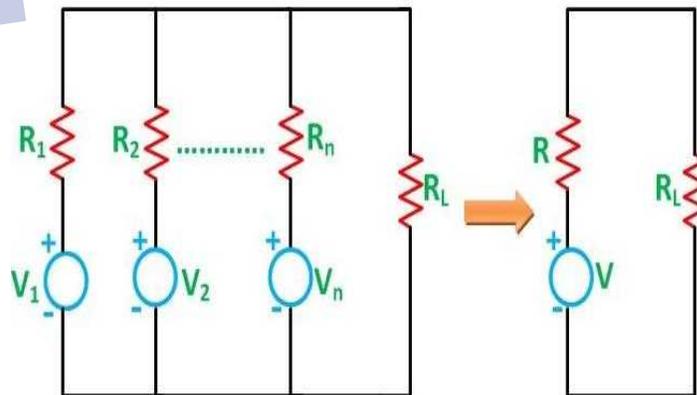
Hence Ammeter reading = $i_a - i = (1.11 - 0.17) = 0.94 A$

Millman’s Theorem:

Millman’s Theorem is a theorem which helps in simplifying electrical networks with a bunch of parallel branches. The utility of this theorem that, any number of parallel voltage sources can be reduced to one equivalent one.

Millman’s Theorem Statement:

The Millman’s Theorem states that – when a number of voltage sources ($V_1, V_2, V_3, \dots, V_n$) are in parallel having internal resistance ($R_1, R_2, R_3, \dots, R_n$) respectively, the arrangement can replace by a single equivalent voltage source V in series with an equivalent series resistance R .

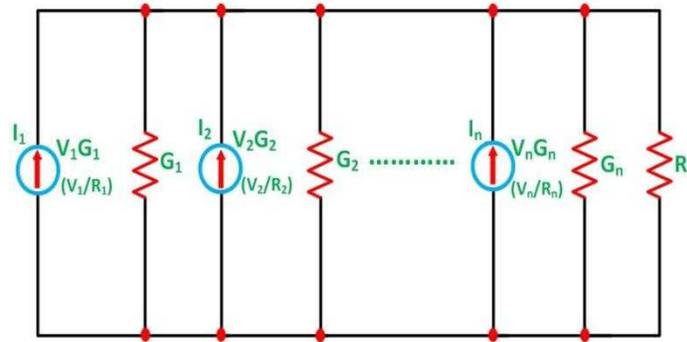


As per Millman’s Theorem,

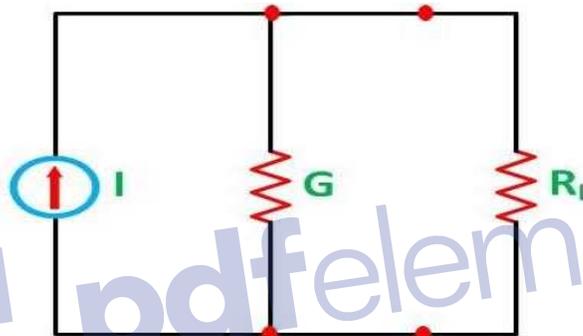
$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \dots \dots \dots \pm V_n G_n}{G_1 + G_2 + \dots \dots \dots + G_n} ;$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \dots \dots \dots + G_n}$$

A DC network of numerous parallel voltage sources with internal resistances supplying power to a load resistance R_L as shown in the figure below.



Let I represent the resultant current of the parallel current sources while G the equivalent conductance as shown in the figure below.



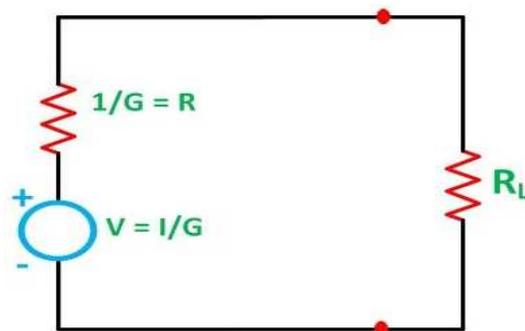
$$I = I_1 + I_2 + I_3 \dots \dots \dots ;$$

$$G = G_1 + G_2 + G_3 \dots \dots \dots \text{(or)}$$

$$I = \frac{\pm I_1 R_1 \pm I_2 R_2 \pm \dots \dots \dots \pm I_n R_n}{R_1 + R_2 + \dots \dots \dots + R_n},$$

$$G = \frac{1}{R} = \frac{1}{R_1 + R_2 + \dots \dots \dots + R_n}$$

The resulting current source is converted to an equivalent voltage source as shown in the fig.



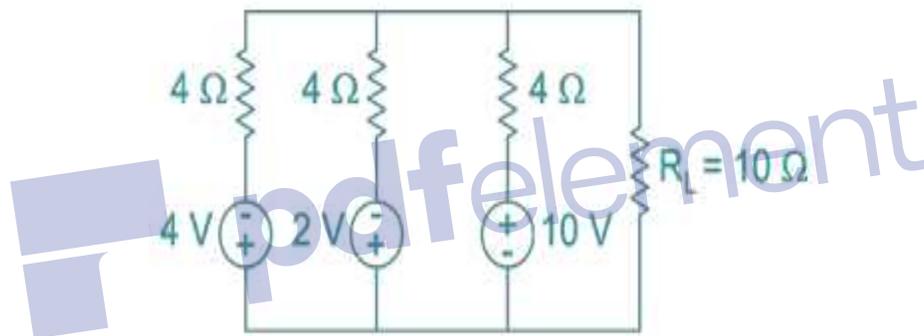
$$V = \frac{I}{G} = \frac{\pm I_1 \pm I_2 \pm \dots \dots \dots \pm I_n}{G_1 + G_2 + \dots \dots \dots + G_n}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \dots \dots \dots + G_n}$$

And as we know, $I = V/R$, and we can also write $R = 1/G$ as $G = 1/R$ So the equation can be written as,

$$V = \frac{\pm \frac{V_1}{R_1} \pm \frac{V_2}{R_2} \pm \dots \dots \dots \pm \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \dots \dots \dots + \frac{1}{R_n}}$$

Example: Find the value of current through R_L using Millman's theorem.?



Solution:

$$\text{Given } R_1 = R_2 = R_3 = 4$$

$$G = G_1 + G_2 + G_3$$

$$G = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$$\therefore R = \frac{1}{G} = \frac{4}{3} \Omega$$

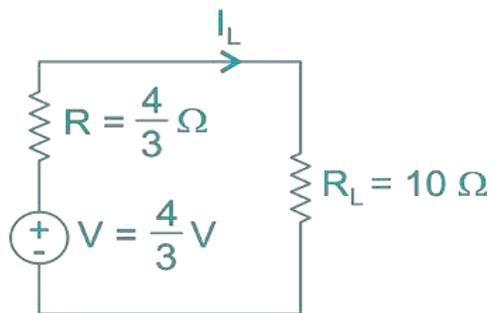
$$V = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3}{G_1 + G_2 + G_3}$$

$$V = \frac{(-4)\frac{1}{4} + (-2)\frac{1}{4} + (10)\frac{1}{4}}{\frac{3}{4}}$$

$$V = \frac{-4 - 2 + 10}{3}$$

$$V = \frac{4}{3}$$

So given circuit becomes,



$$\therefore I_L = \frac{V}{R + R_L} = \frac{\frac{4}{3}}{\frac{4}{3} + 10} = \frac{4}{34} = 117.64 \text{ mA}$$

The current flowing through R_L is **117.64mA**

UNIT-V

MAGNETIC CIRCUITS

- INTRODUCTION
- FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION
- CONCEPT OF SELF & MUTUAL INDUCTANCE
- DOT CONVENTION
- COEFFICIENT OF COUPLING
- COMPOSITE MAGNETIC CIRCUIT
- ANALYSIS OF SERIES AND PARALLEL MAGNETIC CIRCUITS

INTRODUCTION

Although the lines of magnetic flux have no physical existence, they do form a very convenient and useful basis for explaining various magnetic effects and to calculate the magnitudes of various magnetic quantities. The complete closed path followed by any group of magnetic flux lines is referred as magnetic circuit. The lines of magnetic flux never intersect, and each line forms a closed path. Whenever a current is flowing through the coil there will be magnetic flux produced and the path followed by the magnetic flux is known as magnetic circuit. The operation of all the electrical devices like generators, motors, transformers etc. depend upon the magnetism produced by this magnetic circuit. Therefore, to obtain the required characteristics of these devices, their magnetic circuits have to be designed carefully.

Magneto Motive Force (MMF)

The magnetic pressure which sets up or tends to set up magnetic flux in a magnetic circuit is known as MMF.

1. Magneto motive force is the measure of the ability of a coil to produce flux.
2. The magnetic flux is due to the existence of the MMF caused by a current flowing through a coil having no. of turns.
3. ∴ A coil with 'N' turns carrying a current of 'I' amperes represents a magnetic circuit producing an MMF of NI MMF=NI
4. Units of MMF = Ampere turns(AT)

Magnetic Flux:

1. The amount of magnetic lines of force set-up in a magnetic circuit is called magnetic flux.
2. The magnetic flux, that is established in a magnetic circuit is proportional to the MMF and the proportional constant is the reluctance of the magnetic circuit.

Magnetic flux \propto MMF

$$\text{Magnetic flux} = \frac{\text{MMF}}{\text{RELUCTANCE}} = \frac{NI}{S}$$

3. The unit of magnetic flux is Weber.

Reluctance:

1. The opposition offered to the flow of magnetic flux in a magnetic circuit is called reluctance

- Reluctance of a magnetic circuit is defined as the ratio of magneto motive force to the flux established.
- Reluctance depends upon length(l), area of cross-section(a) and permeability of the material that makes up the magnetic circuit. ($S = \frac{l}{\mu a}$)

$$S = \frac{l}{\mu a}$$

$$\text{RELUCTANCE} = \frac{\text{MMF}}{\text{FLUX}}$$

- The unit of reluctance is AT/Wb

Magnetic field strength(H)

- If the magnetic circuit is homogeneous, and of uniform cross-sectional area, the magnetic field strength is defined as the magneto motive force per unit length of magnetic circuit.

$$H = \frac{\text{MMF}}{\text{LENGTH}} = \frac{NI}{l}$$

- The unit of magnetic field strength is AT/m

Magnetic flux density(B)

- The magnetic flux density in any material is defined as the magnetic flux established per unit area of cross-section.

$$B = \frac{\text{FLUX}}{\text{AREA OF CROSS SECTION}} = \frac{\phi}{A}$$

- The unit of magnetic flux density is wb/m² or TESLA

Relative permeability

- It is defined as the ratio of flux density established in magnetic material to the flux density established in air or vacuum for the same magnetic field strength.

INTRODUCTION TO ELECTROMAGNETIC INDUCTION::

When a conductor moves in a magnetic field, an *EMF* is generated; when it carries current in a magnetic field, a force is produced. Both of these effects may be deduced from one of the most fundamental principles of electromagnetism, and they provide the basis for a number of devices in which conductors move freely in a magnetic field. It has already been mentioned that most electrical machines employ a different form of construction.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION:

In 1831, Michael Faraday, an English physicist gave one of the most basic laws of electromagnetism called **Faraday's law of electromagnetic induction**. This law explains the working principle of most of the electrical motors, generators, electrical transformers and inductors. This law shows the relationship between electric circuit and magnetic field.

FARADAY'S FIRST LAW

Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.

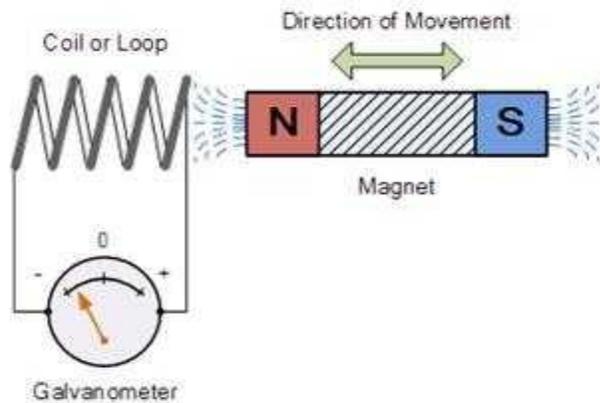
Method to change magnetic field:

- By moving a magnet towards or away from the coil
- By moving the coil into or out of the magnetic field.
- By changing the area of a coil placed in the magnetic field
- By rotating the coil relative to the magnet.

Faraday's Second Law

It states that the magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of number of turns in the coil and flux associated with the coil.

Faraday Law Formula



Consider, a magnet is approaching towards a coil. Here we consider two instants at time T_1 and time T_2 .

Flux linkage with the coil at time,

$$T_1 = N\phi_1 wb$$

Flux linkage with the coil at time,

$$T_2 = N\phi_2 wb$$

Change in flux linkage,

$$N(\phi_2 - \phi_1)$$

Let this change in flux linkage be,

$$\phi = (\phi_2 - \phi_1)$$

So, the Change in flux linkage

$$N\phi$$

Now the rate of change of flux linkage

$$\frac{N\phi}{t}$$

Take derivative above equation we will get the rate of change of flux linkage

$$N \frac{d\phi}{dt}$$

But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.

$$E = N \frac{d\phi}{dt}$$

$$E = - N \frac{d\phi}{dt}$$

Where, flux Φ in Wb = $B \cdot A$

B = magnetic field strength

A = area of the coil

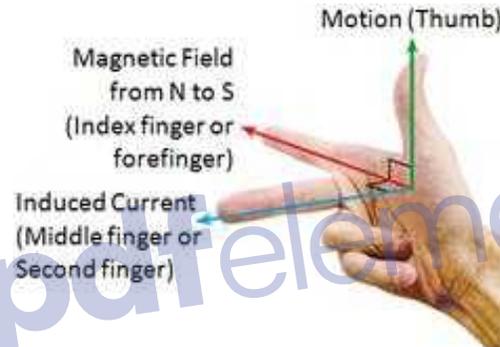
Lenz's law obeys Newton's third law of motion (i.e. to every action there is always an equal and opposite reaction) and the conservation of energy (i.e. energy may neither be created nor destroyed and therefore the sum of all the energies in the system is a constant).

Lenz's law : It states that when an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such, that it produces a current that's magnetic field opposes the change which produces it.

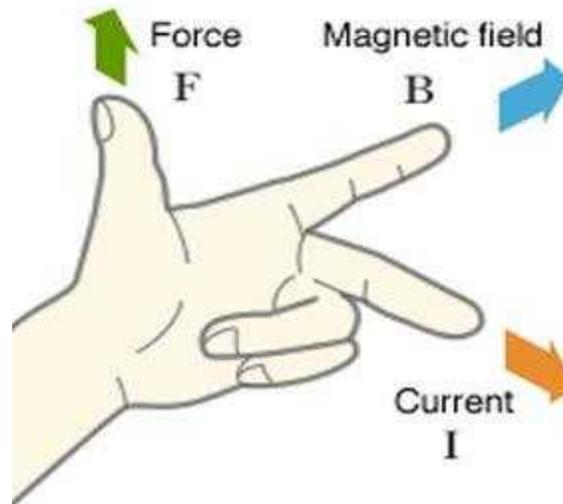
$$E = - N \frac{d\phi}{dt}$$

The negative sign used in Faraday's law of electromagnetic induction, indicates that the induced emf and the change in magnetic flux have opposite signs.

- There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either *Fleming's Right-hand Rule*



- There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either *Fleming's Left-hand Rule*



HOW TO INCREASE EMF INDUCED IN A COIL:

- By increasing the number of turns in the coil i.e. N , from the formulae derived above it is easily seen that if number of turns in a coil is increased, the induced emf also gets increased.
- By increasing magnetic field strength i.e. B surrounding the coil- Mathematically, if magnetic field increases, flux increases and if flux increases emf induced will also get

increased. Theoretically, if the coil is passed through a stronger magnetic field, there will be more lines of force for coil to cut and hence there will be more emf induced.

- By increasing the speed of the relative motion between the coil and the magnet - If the relative speed between the coil and magnet is increased from its previous value, the coil will cut the lines of flux at a faster rate, so more induced emf would be produced.

APPLICATIONS OF FARADAY'S LAW:

Faraday law is one of the most basic and important laws of electromagnetism. This law finds its application in most of the electrical machines, industries and medical field etc.

- Electrical Transformers work on Faraday's law of mutual induction.
- The basic working principle of electrical generator is Faraday's law of electromagnetic induction.
- The Induction cooker is a fastest way of cooking. It also works on principle of mutual induction. When current flows through the coil of copper wire placed below a cooking container, it produces a changing magnetic field. This alternating or changing magnetic field induces an emf and hence the current in the conductive container, and we know that flow of current always produces heat in it.
- Electromagnetic Flow Meter is used to measure velocity of certain fluids. When a magnetic field is applied to electrically insulated pipe in which conducting fluids are flowing, then according to Faraday's law, an electromotive force is induced in it. This induced emf is proportional to velocity of fluid flowing.
- Form the bases of Electromagnetic theory; Faraday's idea of lines of force is used in well known Maxwell's equations. According to Faraday's law, change in magnetic field gives rise to change in electric field and the converse of this is used in Maxwell's equations.
- It is also used in musical instruments like electric guitar, electric violin etc.

SELF INDUCTANCE:

Inductance is the property of electrical circuits containing coils in which a change in the electrical current induces an electromotive force (emf). This value of induced emf opposes the change in current in electrical circuits and electric current 'I' produces a magnetic field which generates magnetic flux acting on the circuit containing coils. **The ratio of the magnetic flux to the current is called the self-inductance.**

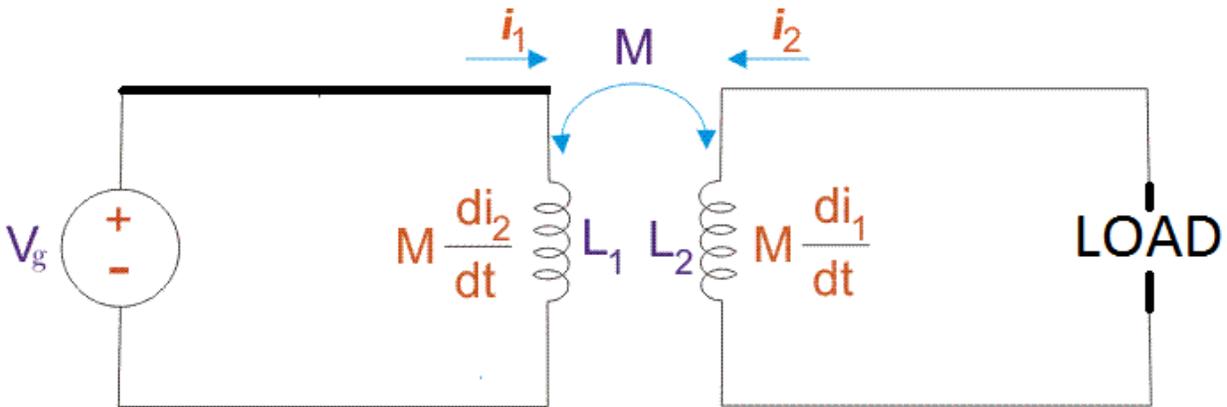
$$L = \frac{\psi = N\phi}{I}$$

The phenomenon of inducing an emf in a coil whenever a current linked with coil changes is called induction. Here units of L are Weber per ampere which is equivalent to Henry.

' ϕ ' denotes the magnetic flux through the area spanned by one loop, 'I' is the current flowing through the coil and N is the number of loops (turns) in the coil.

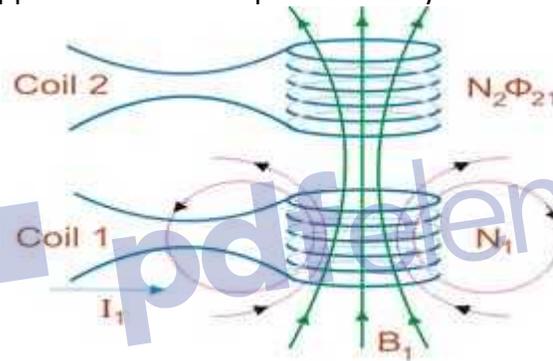
MUTUAL INDUCTANCE:

Mutual Inductance is the ratio between induced Electro Motive Force across a coil to the rate of change of current of another adjacent coil in such a way that two coils are in possibility of flux linkage. Mutual induction is a phenomenon when a coil gets induced in EMF across it due to rate of change current in adjacent coil in such a way that the flux of one coil current gets linkage of another coil. Mutual inductance is denoted as (M), it is called co-efficient of Mutual Induction between two coils



Mutual inductance for two coils gives the same value when they are in mutual induction with each other. Induction in one coil due to its own rate of change of current is called self inductance (L), but due to rate of change of current of adjacent coil it gives **mutual inductance (M)**

From the above figure, first coil carries current i_1 and its self inductance is L_1 . Along with its self inductance it has to face mutual induction due to rate of change of current i_2 in the second coil. Same case happens in the second coil also. Dot convention is used to mark the polarity of the mutual induction. Suppose two coils are placed nearby



Coil 1 carries I_1 current having N_1 number of turn. Now the flux density created by the coil 1 is B_1 . Coil 2 with N_2 number of turn gets linked with this flux from coil 1. So flux linkage in coil 2 is N_2 .

ϕ_{21} [ϕ_{21} is called leakage flux in coil 2 due to coil 1].

$$\varepsilon_2 = -N_2 \cdot \frac{d\phi_{21}}{dt} \text{ volt.}$$

$$\text{Again, } \varepsilon_2 = -M_{21} \cdot \frac{di_1}{dt} \text{ volt.}$$

Now it can be written from these equations,

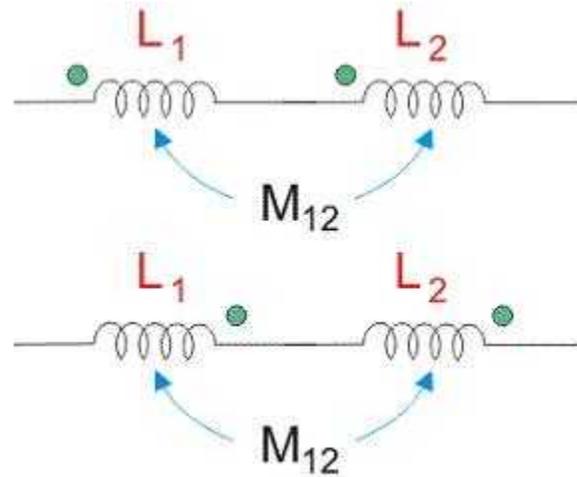
$$M_{21} = \frac{\phi_{21} N_2}{I_1}$$

DOT CONVENTION:

- Dot convention is used to determine the polarity of a magnetic coil in respect of other magnetic coil.
- Dot convention is normally used to determine the total or equivalent inductance (L_{eq}).

SERIES AIDING:

- Suppose two coils are in series with same place dot.
- When 2 dots are at the same place of both inductors(while at entering place or leaving place)as shown in below figure i.e. the total mutual inductance gets aided(added)

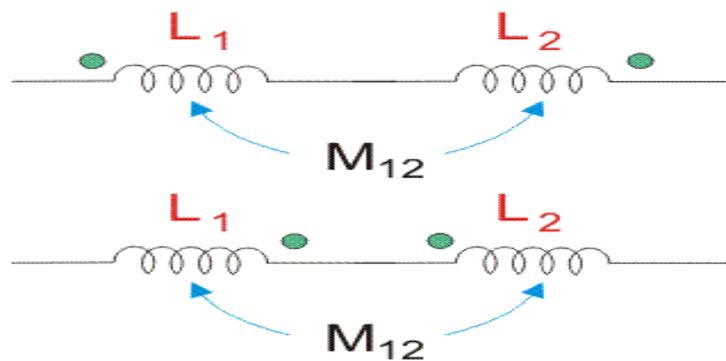


Mutual inductance between them is positive.

$$\text{So, } L_{eq} = L_1 + L_2 + 2M_{12}$$

SERIES OPPOSING:

- Suppose two coils are in series with opposite place dot.
- When 2 dots are at the opposite place of both inductors(while one at entering place and other at leaving place)as shown in below figure i.e. the total mutual inductance gets differed

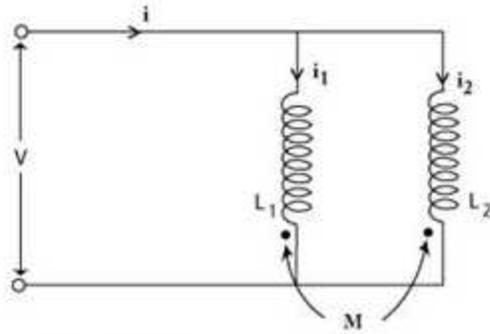


Mutual inductance between them is negative.

$$\text{So, } L_{eq} = L_1 + L_2 - 2M_{12}$$

PARALLEL AIDING:

- Suppose two coils are in parallel with same place dot.
- When 2 dots are at the same place of both inductors(while at entering place or leaving place)as shown in below figure i.e. the total mutual inductance gets aided(added)



$$V = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix} = \begin{bmatrix} V \\ V \end{bmatrix}$$

$$\Delta = \begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix} = L_1 L_2 - M^2$$

$$\frac{di_1}{dt} = \frac{\begin{vmatrix} V & M \\ V & L_2 \end{vmatrix}}{\Delta} = \frac{V(L_2 - M)}{\Delta}, \quad \frac{di_2}{dt} = \frac{\begin{vmatrix} L_1 & V \\ M & V \end{vmatrix}}{\Delta} = \frac{V(L_1 - M)}{\Delta}$$

From the above figure,

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{V(L_2 - M)}{\Delta} + \frac{V(L_1 - M)}{\Delta} = \frac{V(L_1 + L_2 - 2M)}{\Delta} = \frac{V(L_1 + L_2 - 2M)}{L_1 L_2 - M^2}$$

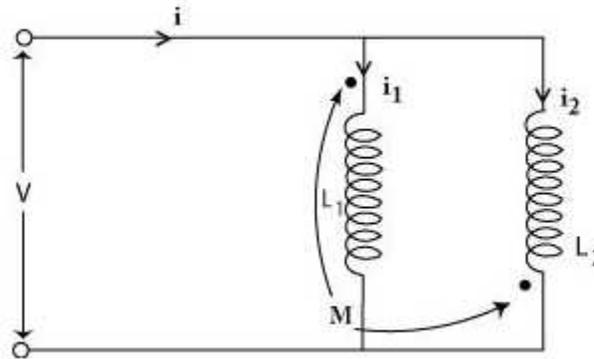
$$V = \frac{L_1 L_2 - M^2}{(L_1 + L_2 - 2M)} \frac{di}{dt}$$

Therefore total inductance is given by,

$$L_{eq} = \frac{L_1 L_2 - M^2}{(L_1 + L_2 - 2M)}$$

PARALLEL OPPOSING:

- Suppose two coils are in parallel with opposite place dot.
- When 2 dots are at the opposite place of both inductors(while one at entering place and other at leaving place)as shown in below figure i.e. the total mutual inductance gets differed



$$V = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$V = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

$$\begin{bmatrix} L_1 & -M \\ -M & L_2 \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix} = \begin{bmatrix} V \\ V \end{bmatrix}$$

$$\Delta = \begin{vmatrix} L_1 & -M \\ -M & L_2 \end{vmatrix} = L_1 L_2 - M^2$$

$$\frac{di_1}{dt} = \frac{\begin{vmatrix} V & -M \\ V & L_2 \end{vmatrix}}{\Delta} = \frac{V(L_2 + M)}{\Delta}, \quad \frac{di_2}{dt} = \frac{\begin{vmatrix} L_1 & V \\ -M & V \end{vmatrix}}{\Delta} = \frac{V(L_1 + M)}{\Delta}$$

From the above figure,

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{V(L_2 + M)}{\Delta} + \frac{V(L_1 + M)}{\Delta} = \frac{V(L_1 + L_2 + 2M)}{\Delta} = \frac{V(L_1 + L_2 + 2M)}{L_1 L_2 - M^2}$$

$$V = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \frac{di}{dt}$$

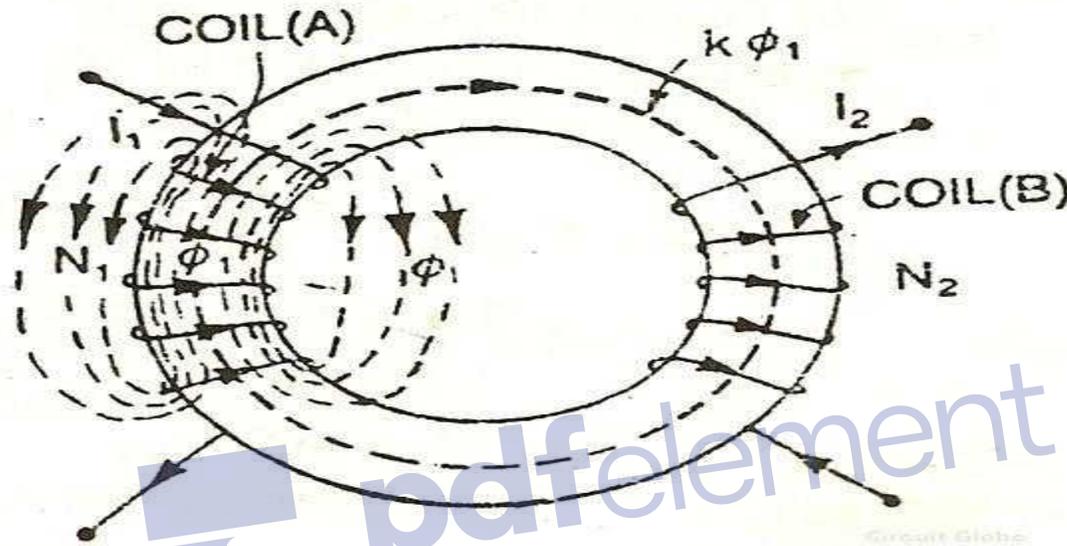
Therefore total inductance is given by,

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

COEFFICIENT OF COUPLING:

The fraction of magnetic flux produced by the current in one coil that links with the other coil is called **coefficient of coupling** between the two coils. It is denoted by (k).

Two coils are taken coil A and coil B, when current flows through one coil it produces flux; the whole flux may not link with the other coil coupled, and this is because of leakage flux by a fraction (k) known as **Coefficient of Coupling**.



$k=1$ when the flux produced by one coil completely links with the other coil and is called magnetically tightly coupled.

$k=0$ when the flux produced by one coil does not link at all with the other coil and thus the coils are said to be magnetically isolated.

DERIVATION:

Consider two magnetic coils A and B.
When current I_1 flows through coil A.

$$L_1 = \frac{N_1 \phi_1}{I_1} \text{ and } M = \frac{N_2 \phi_{12}}{I_1} \dots \dots \dots (1) \text{ as } (\phi_{12} = k\phi_1)$$

Considering coil B in which current I_2 flows

$$L_2 = \frac{N_2 \phi_2}{I_2} \text{ and } M = \frac{N_1 \phi_{21}}{I_2} = \frac{N_1 k \phi_2}{I_2} \dots \dots \dots (2) \text{ as } (\phi_2 = k\phi_2)$$

Multiplying equation (1) and (2)

$$M \times M = \frac{N_2 k \phi_1}{I_1} \times \frac{N_1 k \phi_2}{I_2}$$

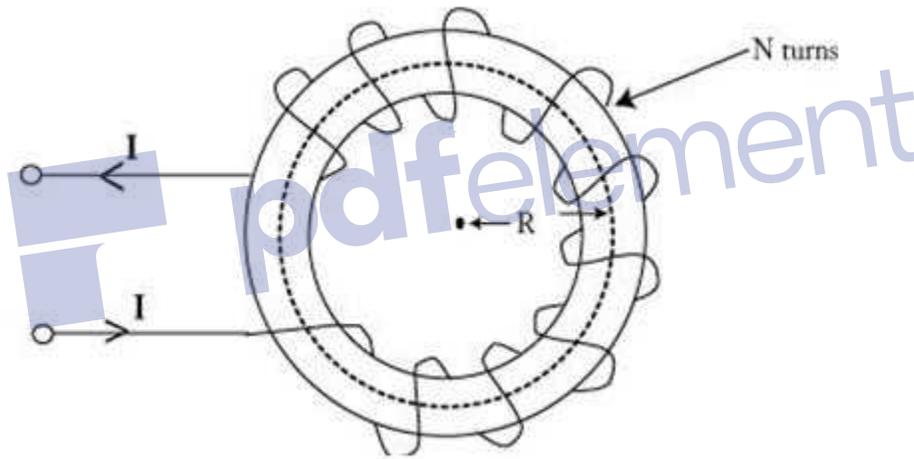
$$M^2 = k^2 \frac{N_1 \phi_1}{I_1} \times \frac{N_2 \phi_2}{I_2} = k^2 L_1 L_2$$

$$M = \sqrt{L_1 L_2} \dots \dots \dots (A)$$

The above equation (A) shows the relationship between mutual inductance and self inductance between two the coils

SERIES MAGNETIC CIRCUIT:

- A series magnetic circuit is analogous to a series electric circuit. A magnetic circuit is said to be series, if the same flux is flowing through all the elements connected in a magnetic circuit. Consider a circular ring having a magnetic path of 'l' meters, area of cross section 'a' m² with a mean radius of 'R' meters having a coil of 'N' turns carrying a current of 'I' amperes wound uniformly as shown in below fig



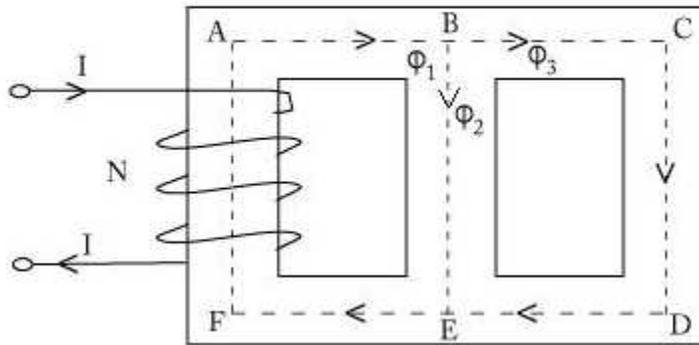
The flux produced by the circuit is given by

$$\text{Magnetic flux} = \frac{\text{MMF}}{\text{RELUCTANCE}} = \frac{NI}{S} = \frac{NI}{l/\mu a}$$

In the above equation NI is the MMF of the magnetic circuit, which is analogous to EMF in the electrical circuit.

PARALLEL MAGNETIC CIRCUIT

- A magnetic circuit which has more than one path for magnetic flux is called a parallel magnetic circuit. It can be compared with a parallel electric circuit which has more than one path for electric current. The concept of parallel magnetic circuit is illustrated in fig. 2. Here a coil of 'N' turns wound on limb 'AF' carries a current of 'I' amperes. The magnetic flux ' ϕ_1 ' set up by the coil divides at 'B' into two paths namely
 - Magnetic flux passes ' ϕ_2 ' along the path 'BE'
 - Magnetic flux passes ' ϕ_3 ' along the path 'BCDE' i.e. $\phi_1 = \phi_2 + \phi_3$



The magnetic paths 'BE' and 'BCDE' are in parallel and form a parallel magnetic circuit. The AT required for this parallel circuit is equal to AT required for any one of the paths. Let S_1 = reluctance of path EFAB

Let, S_1 = reluctance of path EFAB

S_2 = reluctance of path BE

S_3 = reluctance of path BCDE

Total MMF = MMF for path EFAB + MMF for path BE or path BCDE

$$NI = \Phi_1 S_1 + \Phi_2 S_2 = \Phi_1 S_1 + \Phi_3 S_3$$

COMPOSITE MAGNETIC CIRCUIT:

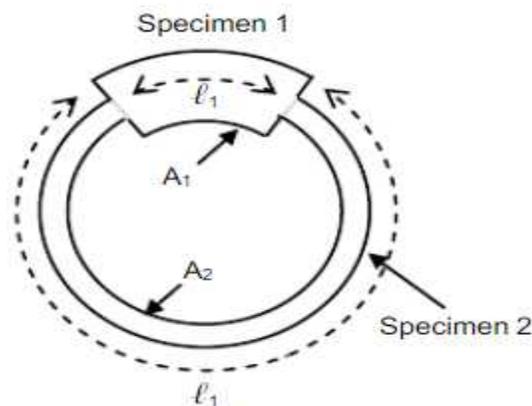
Consider a magnetic circuit which consists of two specimens of iron arranged as shown in figure. Let ℓ_1 and ℓ_2 be the mean lengths of specimen 1 and specimen 2 in meters, A_1 and A_2 be their respective cross sectional areas in square meters, and μ_1 and μ_2 be their respective relative permeability's.

The reluctance of specimen 1 is given as

$$S_1 = \frac{\ell_1}{\mu_0 \mu_1 A_1} \quad (AT/Wb)$$

and that for specimen 2 is

$$S_2 = \frac{\ell_2}{\mu_0 \mu_2 A_2} \quad (AT/Wb)$$

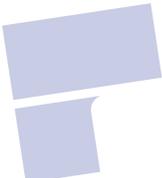


If a coil of N turns carrying a current I is wound on the specimen 1 and if the magnetic flux is assumed to be confined to iron core then the total reluctance is given by the sum of the individual reluctances S_1 and S_2 . This is equivalent to adding the resistances of a series circuit. Thus the total reluctance is given by

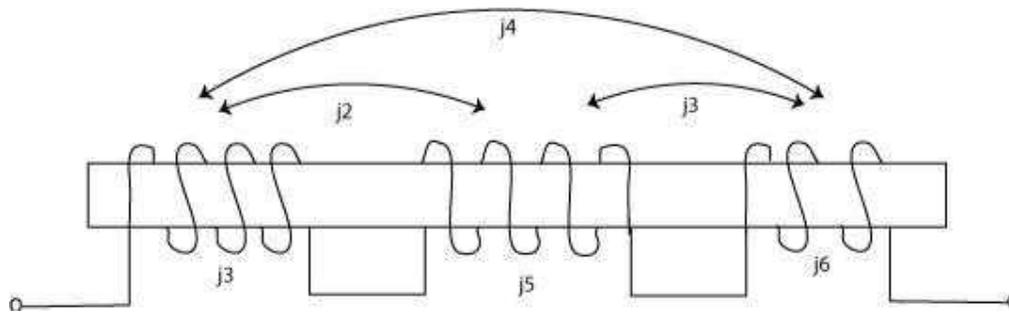
$$S = S_1 + S_2 = \frac{\ell_1}{\mu_0 \mu_1 A_1} + \frac{\ell_2}{\mu_0 \mu_2 A_2} \quad AT/Wb$$

And the total flux is given by

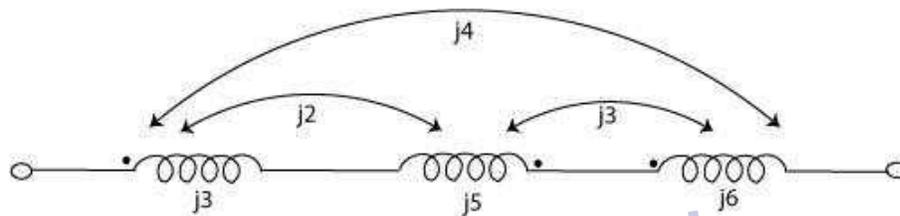
$$\Phi = \frac{\text{mmf}}{S} = \frac{NI}{\frac{\ell_1}{\mu_0 \mu_1 A_1} + \frac{\ell_2}{\mu_0 \mu_2 A_2}} \quad \left(\frac{AT}{(AT/Wb)} \Rightarrow Wb \right)$$

 pdfelement

Problem : Sketch the dotted equivalent circuit for the coupled coil shown in the fig. and find the equivalent inductive?



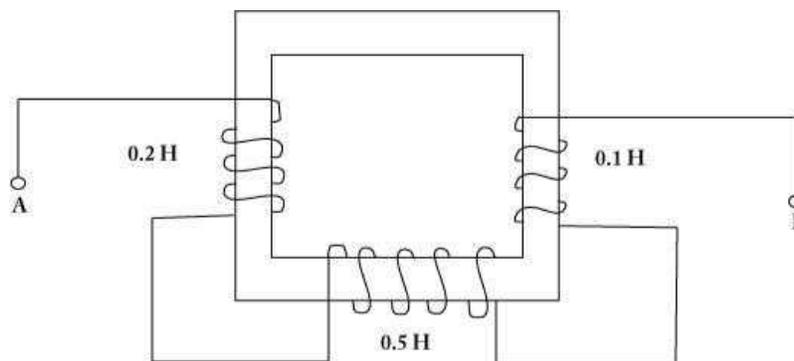
Solution: The dotted equivalent circuit is



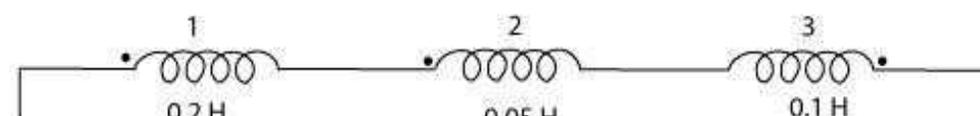
The equivalent inductive reactance is

$$j X_{eq} = j3 + j5 + j6 - 2 \times j2 - 2 \times j3 + 2 \times j4 = j14 - j2 = j12$$

Problem: Sketch the dotted equivalent circuit for the coupled coils shown in figure and find the equivalent inductance at the terminals AB. All coupling coefficients are 0.5.



Solution: The dotted equivalent circuit is



$$M_{12} = 0.5\sqrt{0.2 \times 0.05} = 0.05$$

$$= \sqrt{\quad \times \quad} =$$

$$M_{23} = 0.5\sqrt{0.05 \times 0.1} = 0.0035$$

$$= \sqrt{\quad \times \quad} =$$

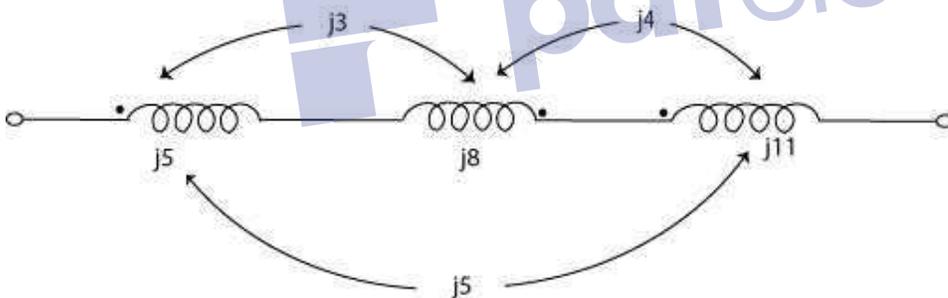
$$M_{31} = 0.5\sqrt{0.2 \times 0.1} = 0.0707$$

Equivalent inductance between terminals A and B $L_{AB} = 0.2 + 0.05 + 0.1 + 2(0.05 - 0.035 - 0.0707)$

$$= 0.2386\text{H}$$

Problem: For the given circuit as shown in figure

- Find the equivalent reactance
- Draw the transformer coupling circuit

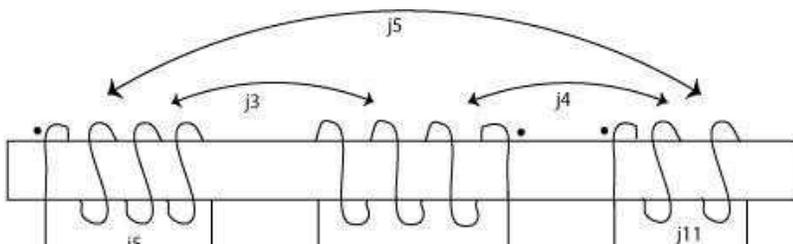


Solution

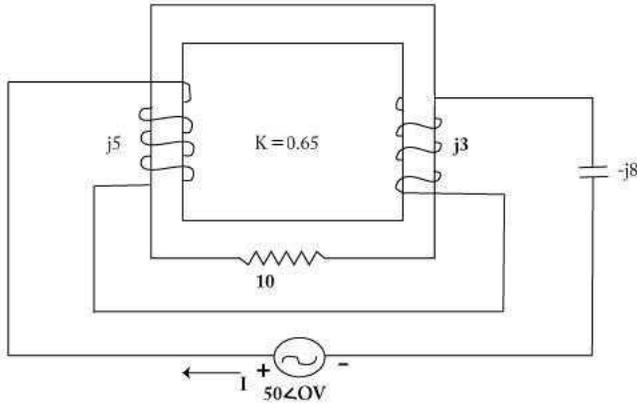
- The equivalent reactance is

$$j X_{eq} = j5 + j8 + j11 + 2(-j3 - j4 + j5) = j24 - j4 = j20$$

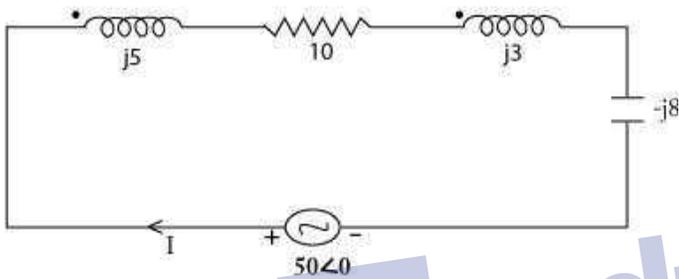
- The T/F coupling circuit is



Problem: Sketch the dotted equivalent circuit for the coupled coils shown in figure and find the current I ?



Solution: The dotted equivalent circuit is



$$jX_M = jK \sqrt{X_{L1} X_{L2}} = j0.65 \sqrt{5 \cdot 3} = j2.5$$

By applying KVL to the circuit

$$10I - j8I + (j5 + j3 + 2 \times j2.5)I = 50$$

$$I(10 - j8 + j3) = 50$$

$$I = 4.47 \text{ at an angle of } 26.56^\circ \text{ A}$$